CHAPTER2: ELECTROSTATIC POTENTIAL AND CAPACITANCE

NCERT EXERCISES

2.1 Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

2.2 A regular hexagon of side 10 cm has a charge 5 µC at each of its vertices. Calculate the potential at the centre of the hexagon.

2.3 Two charges 2 µC and -2 µC are placed at points A and B 6 cm apart.
(a) Identify an equipotential surface of the system.
(b) What is the direction of the electric field at every point on this surface?

2.4 A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7}C distributed uniformly on its surface. What is the electric field
(a) inside the sphere
(b) just outside the sphere
(c) at a point 18 cm from the centre of the sphere?

2.5 A parallel plate capacitor with air between the plates has a capacitance of 8 pF ($1pF = 10^{-12}$ F). What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?

2.6 Three capacitors each of capacitance 9 pF are connected in series.
(a) What is the total capacitance of the combination?
(b) What is the potential difference across each capacitor if the combination is connected to a 120 V supply?

2.7 Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel.
(a) What is the total capacitance of the combination?
(b) Determine the charge on each capacitor if the combination is connected to a 100 V supply.

2.8 In a parallel plate capacitor with air between the plates, each plate has an area of 6×10^{-3} m^2 and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?

2.9 Explain what would happen if in the capacitor given in Exercise 2.8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates,
(a) while the voltage supply remained connected.
(b) after the supply was disconnected.

2.10 A 12pF capacitor is connected to a 50V battery. How much electrostatic energy is stored in the capacitor?

2.11 A 600pF capacitor is charged by a 200V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?

ADDITIONAL EXERCISES

2.12 A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of -2×10^{-9} C from a point P (0, 0, 3 cm) to a point Q (0, 4 cm, 0), via a point R (0, 6 cm, 9 cm).

2.13 A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube.

2.14 Two tiny spheres carrying charges 1.5 μC and 2.5 μC are located 30 cm apart. Find the potential and electric field:
(a) at the mid-point of the line joining the two charges, and
(b) at a point 10 cm from this midpoint in a plane normal to the line and passing through the mid-point.

2.15 A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q.
(a) A charge q is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell?
(b) Is the electric field inside a cavity (with no charge) zero, even if the shell is not spherical, but has any irregular shape? Explain.

2.16 (a) Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by

$$(\mathbf{E}_2 - \mathbf{E}_1) \cdot \hat{\mathbf{n}} = \frac{\sigma}{\varepsilon_0}$$

where $\hat{\mathbf{n}}$ is a unit vector normal to the surface at a point and σ is the surface charge density at that point. (The direction of $\hat{\mathbf{n}}$ is from side 1 to side 2.) Hence show that just outside a conductor, the electric field is $\sigma \, \hat{\mathbf{n}} /\varepsilon_0$.
(b) Show that the tangential component of electrostatic field is continuous from one side of a charged surface to another. [Hint: For (a), use Gauss's law. For (b), use the fact that work done by electrostatic field on a closed loop is zero.]

2.17 A long charged cylinder of linear charged density λ is surrounded by a hollow co-axial conducting cylinder. What is the electric field in the space between the two cylinders?

2.18 In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 Å:
(a) Estimate the potential energy of the system in eV, taking the zero of the potential energy at infinite separation of the electron from proton.
(b) What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (a)?
(c) What are the answers to (a) and (b) above if the zero of potential energy is taken at 1.06 Å separation?

2.19 If one of the two electrons of a H_2 molecule is removed, we get a hydrogen molecular ion H_2^+. In the ground state of an H_2^+, the two protons are separated by roughly 1.5 Å, and the electron is roughly 1 Å from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy.

2.20 Two charged conducting spheres of radii a and b are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions.

2.21 Two charges $-q$ and $+q$ are located at points $(0, 0, -a)$ and $(0, 0, a)$,
✳✳ respectively.
(a) What is the electrostatic potential at the points $(0, 0, z)$ and $(x, y, 0)$?
(b) Obtain the dependence of potential on the distance r of a point from the origin when $r/a \gg 1$.
(c) How much work is done in moving a small test charge from the point $(5,0,0)$ to $(-7,0,0)$ along the x-axis? Does the answer change if the path of the test charge between the same points is not along the x-axis?

2.22 Figure 2.34 shows a charge array known as an *electric quadrupole*.
✳✳ For a point on the axis of the quadrupole, obtain the dependence of potential on r for $r/a \gg 1$, and contrast your results with that due to an electric dipole and an electric monopole (i.e., a single charge).
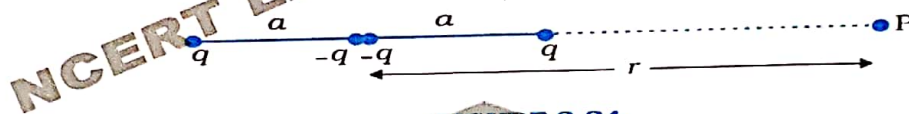

FIGURE 2.34

2.23 An electrical technician requires a capacitance of 2 µF in a circuit across a potential difference of 1 kV. A large number of 1 µF capacitors are available to him each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires the minimum number of capacitors.

2.24 What is the area of the plates of a 2 F parallel plate capacitor, given that the separation between the plates is 0.5 cm? [You will realise from your answer why ordinary capacitors are in the range of µF or less. However, electrolytic capacitors do have a much larger capacitance (0.1 F) because of very minute separation between the conductors.]

2.25 Obtain the equivalent capacitance of the network in Fig. 2.35. For a
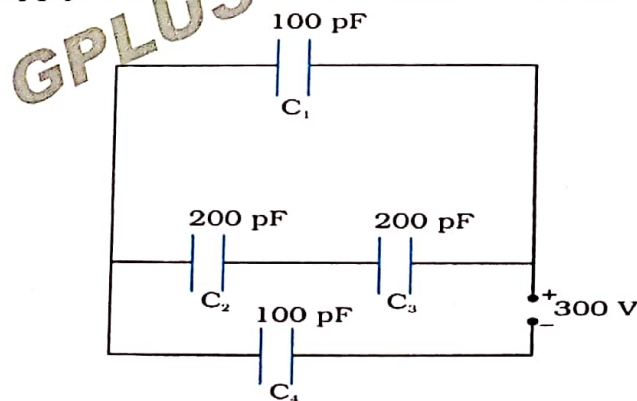**✳✳** 300 V supply, determine the charge and voltage across each capacitor.

FIGURE 2.35

2.26 The plates of a parallel plate capacitor have an area of 90 cm² each
✳ and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply.
(a) How much electrostatic energy is stored by the capacitor?
(b) View this energy as stored in the electrostatic field between the plates, and obtain the energy per unit volume u. Hence arrive at a relation between u and the magnitude of electric field E between the plates.

2.27 A 4 µF capacitor is charged by a 200 V supply. It is then disconnected from the supply, and is connected to another uncharged 2 µF capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation?

2.28 Show that the force on each plate of a parallel plate capacitor has a magnitude equal to ($\frac{1}{2}$) QE, where Q is the charge on the capacitor, and E is the magnitude of electric field between the plates. Explain the origin of the factor $\frac{1}{2}$.

2.29 A spherical capacitor consists of two concentric spherical conductors, held in position by suitable insulating supports (Fig. 2.36). Show
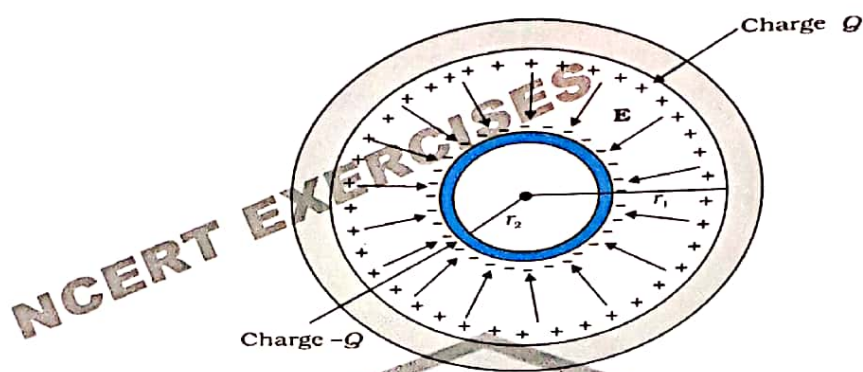

Charge Q
Charge $-Q$

FIGURE 2.36

that the capacitance of a spherical capacitor is given by

$$C = \frac{4\pi\varepsilon_0\, r_1 r_2}{r_1 - r_2}$$

where r_1 and r_2 are the radii of outer and inner spheres, respectively.

2.30 A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5 μC. The space between the concentric spheres is filled with a liquid of dielectric constant 32.
  (a) Determine the capacitance of the capacitor.
  (b) What is the potential of the inner sphere?
  (c) Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Explain why the latter is much smaller.

2.31 Answer carefully:
  (a) Two large conducting spheres carrying charges Q_1 and Q_2 are brought close to each other. Is the magnitude of electrostatic force between them exactly given by $Q_1 Q_2/4\pi\varepsilon_0 r^2$, where r is the distance between their centres?
  (b) If Coulomb's law involved $1/r^3$ dependence (instead of $1/r^2$), would Gauss's law be still true ?
  (c) A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the field line passing through that point?
  (d) What is the work done by the field of a nucleus in a complete circular orbit of the electron? What if the orbit is elliptical?
  (e) We know that electric field is discontinuous across the surface of a charged conductor. Is electric potential also discontinuous there?
  (f) What meaning would you give to the capacitance of a single conductor?
  (g) Guess a possible reason why water has a much greater dielectric constant (= 80) than say, mica (= 6).

2.32 A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 µC. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (i.e., bending of field lines at the ends).

2.33 A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about 10^7 Vm^{-1}. (Dielectric strength is the maximum electric field a material can tolerate without breakdown, i.e., without starting to conduct electricity through partial ionisation.) For safety, we should like the field never to exceed, say 10% of the dielectric strength. What minimum area of the plates is required to have a capacitance of 50 pF?

2.34 Describe schematically the equipotential surfaces corresponding to
(a) a constant electric field in the z-direction,
(b) a field that uniformly increases in magnitude but remains in a constant (say, z) direction,
(c) a single positive charge at the origin, and
(d) a uniform grid consisting of long equally spaced parallel charged wires in a plane.

2.35 In a Van de Graaff type generator a spherical metal shell is to be a 15×10^6 V electrode. The dielectric strength of the gas surrounding the electrode is 5×10^7 Vm^{-1}. What is the minimum radius of the spherical shell required? (You will learn from this exercise why one cannot build an electrostatic generator using a very small shell which requires a small charge to acquire a high potential.)

2.36 A small sphere of radius r_1 and charge q_1 is enclosed by a spherical shell of radius r_2 and charge q_2. Show that if q_1 is positive, charge will necessarily flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge q_2 on the shell is.

2.37 Answer the following:
(a) The top of the atmosphere is at about 400 kV with respect to the surface of the earth, corresponding to an electric field that decreases with altitude. Near the surface of the earth, the field is about 100 Vm^{-1}. Why then do we not get an electric shock as we step out of our house into the open? (Assume the house to be a steel cage so there is no field inside!)
(b) A man fixes outside his house one evening a two metre high insulating slab carrying on its top a large aluminium sheet of area 1m^2. Will he get an electric shock if he touches the metal sheet next morning?
(c) The discharging current in the atmosphere due to the small conductivity of air is known to be 1800 A on an average over the globe. Why then does the atmosphere not discharge itself completely in due course and become electrically neutral? In other words, what keeps the atmosphere charged?
(d) What are the forms of energy into which the electrical energy of the atmosphere is dissipated during a lightning?
(Hint: The earth has an electric field of about 100 Vm^{-1} at its surface in the downward direction, corresponding to a surface charge density = -10^{-9} C m^{-2}. Due to the slight conductivity of the atmosphere up to about 50 km (beyond which it is good conductor), about + 1800 C is pumped every second into the earth as a whole. The earth, however, does not get discharged since thunderstorms and lightning occurring continually all over the globe pump an equal amount of negative charge on the earth.)