NCERT EXERCISES

You may find the following data useful in solving the exercises:

$e = 1.6\times10^{-19}C$ $\quad N = 6.023\times10^{23}$ per mole

$1/(4\pi\varepsilon_0) = 9\times10^9\ N\ m^2/C^2$ $\quad k = 1.381\times10^{-23}J\ ^0K^{-1}$

$1\ MeV = 1.6\times10^{-13}J$ $\quad 1\ u = 931.5\ MeV/c^2$

$1\ year = 3.154\times10^7\ s$

$m_H = 1.007825\ u$ $\quad m_n = 1.008665\ u$

$m(^4_2He) = 4.002603\ u$ $\quad m_e = 0.000548\ u$

13.1 (a) Two stable isotopes of lithium ^6_3Li and ^7_3Li have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512 u and 7.01600 u, respectively. Find the atomic mass of lithium.

(b) Boron has two stable isotopes, $^{10}_5B$ and $^{11}_5B$. Their respective masses are 10.01294 u and 11.00931 u, and the atomic mass of boron is 10.811 u. Find the abundances of $^{10}_5B$ and $^{11}_5B$.

13.2 The three stable isotopes of neon: $^{20}_{10}Ne$, $^{21}_{10}Ne$ and $^{22}_{10}Ne$ have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u, respectively. Obtain the average atomic mass of neon.

13.3 Obtain the binding energy (in MeV) of a nitrogen nucleus $(^{14}_7N)$, given $m\ (^{14}_7N)=14.00307\ u$

13.4 Obtain the binding energy of the nuclei $^{56}_{26}Fe$ and $^{209}_{83}Bi$ in units of MeV from the following data:

$m\ (^{56}_{26}Fe) = 55.934939\ u$ $\quad m\ (^{209}_{83}Bi) = 208.980388\ u$

13.5 A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of $^{63}_{29}Cu$ atoms (of mass 62.92960 u).

13.6 Write nuclear reaction equations for

(i) α-decay of $^{226}_{88}Ra$ (ii) α-decay of $^{242}_{94}Pu$

(iii) β^--decay of $^{32}_{15}P$ (iv) β^--decay of $^{210}_{83}Bi$

(v) β^+-decay of $^{11}_6C$ (vi) β^+-decay of $^{97}_{43}Tc$

(vii) Electron capture of $^{120}_{54}Xe$

13.7 A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to a) 3.125%, b) 1% of its original value?

13.8 The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive $^{14}_6C$ present with the stable carbon isotope $^{12}_6C$. When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of $^{14}_6C$, and the measured activity, the age of the specimen can be approximately estimated. This is the principle of $^{14}_6C$ dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilisation.

13.9 Obtain the amount of $^{60}_{27}Co$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of $^{60}_{27}Co$ is 5.3 years.

13.10 The half-life of $^{90}_{38}Sr$ is 28 years. What is the disintegration rate of 15 mg of this isotope?

13.11 Obtain approximately the ratio of the nuclear radii of the gold isotope $^{197}_{79}Au$ and the silver isotope $^{107}_{47}Ag$.

13.12 Find the Q-value and the kinetic energy of the emitted α-particle in the α-decay of (a) $^{226}_{88}Ra$ and (b) $^{220}_{86}Rn$.

Given m ($^{226}_{88}Ra$) = 226.02540 u, m ($^{222}_{86}Rn$) = 222.01750 u,

m ($^{222}_{86}Rn$) = 220.01137 u, m ($^{216}_{84}Po$) = 216.00189 u.

13.13 The radionuclide ^{11}C decays according to

$$^{11}_{6}C \rightarrow ^{11}_{5}B + e^{+} + \nu: \quad T_{1/2} = 20.3 \text{ min}$$

The maximum energy of the emitted positron is 0.960 MeV.

Given the mass values:

m ($^{11}_{6}C$) = 11.011434 u and m ($^{11}_{6}B$) = 11.009305 u,

calculate Q and compare it with the maximum energy of the positron emitted.

13.14 The nucleus $^{23}_{10}Ne$ decays by β^{-} emission. Write down the β-decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:

m ($^{23}_{10}Ne$) = 22.994466 u

m ($^{23}_{11}Na$) = 22.089770 u.

13.15 The Q value of a nuclear reaction $A + b \rightarrow C + d$ is defined by

$Q = [\, m_A + m_b - m_C - m_d]c^2$

where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.

(i) $^{1}_{1}H + ^{3}_{1}H \rightarrow ^{2}_{1}H + ^{2}_{1}H$

(ii) $^{12}_{6}C + ^{12}_{6}C \rightarrow ^{20}_{10}Ne + ^{4}_{2}He$

Atomic masses are given to be

m ($^{2}_{1}H$) = 2.014102 u

m ($^{3}_{1}H$) = 3.016049 u

m ($^{12}_{6}C$) = 12.000000 u

m ($^{20}_{10}Ne$) = 19.992439 u

13.16 Suppose, we think of fission of a $^{56}_{26}Fe$ nucleus into two equal fragments, $^{28}_{13}Al$. Is the fission energetically possible? Argue by working out Q of the process. Given m ($^{56}_{26}Fe$) = 55.93494 u and m ($^{28}_{13}Al$) = 27.98191 u.

13.17 The fission properties of $^{239}_{94}Pu$ are very similar to those of $^{235}_{92}U$. The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure $^{239}_{94}Pu$ undergo fission?

13.18 A 1000 MW fission reactor consumes half of its fuel in 5.00 y. How much $^{235}_{92}U$ did it contain initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission of $^{235}_{92}U$ and that this nuclide is consumed only by the fission process.

13.19 How long can an electric lamp of 100W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as

$$^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He + n + 3.27 \text{ MeV}$$

13.20 Calculate the height of the potential barrier for a head on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)

13.21 From the relation $R = R_0 A^{1/3}$, where R_0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e. independent of A).

13.22 For the β^+ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K–shell, is captured by the nucleus and a neutrino is emitted).

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$e^+ + {}^A_Z X \rightarrow {}_{Z-1}^{\ A}Y + \nu$

Show that if β^+ emission is energetically allowed, electron capture is necessarily allowed but not vice–versa.

ADDITIONAL EXERCISES

13.23 In a periodic table the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative natural abundance on earth. The three isotopes and their masses are ${}^{24}_{12}Mg$ (23.98504u), ${}^{25}_{12}Mg$ (24.98584u) and ${}^{26}_{12}Mg$ (25.98259u). The natural abundance of ${}^{24}_{12}Mg$ is 78.99% by mass. Calculate the abundances of other two isotopes.

13.24 The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei ${}^{41}_{20}Ca$ and ${}^{27}_{13}Al$ from the following data:

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$m({}^{40}_{20}Ca) = 39.962591$ u

$m({}^{41}_{20}Ca) = 40.962278$ u

$m({}^{26}_{13}Al) = 25.986895$ u

$m({}^{27}_{13}Al) = 26.981541$ u

13.25 A source contains two phosphorous radio nuclides ${}^{32}_{15}P$ ($T_{1/2}$ = 14.3d) and ${}^{33}_{15}P$ ($T_{1/2}$ = 25.3d). Initially, 10% of the decays come from ${}^{33}_{15}P$. How long one must wait until 90% do so?

13.26 Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α-particle. Consider the following decay processes:

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$${}^{223}_{88}Ra \rightarrow {}^{209}_{82}Pb + {}^{14}_{6}C$$

$${}^{223}_{88}Ra \rightarrow {}^{219}_{86}Rn + {}^{4}_{2}He$$

Calculate the Q-values for these decays and determine that both are energetically allowed.

13.27 Consider the fission of ${}^{238}_{92}U$ by fast neutrons. In one fission event, no neutrons are emitted and the final end products, after the beta decay of the primary fragments, are ${}^{140}_{58}Ce$ and ${}^{99}_{44}Ru$. Calculate Q for this fission process. The relevant atomic and particle masses are

$m({}^{238}_{92}U)$ =238.05079 u

$m({}^{140}_{58}Ce)$ =139.90543 u

$m({}^{99}_{44}Ru)$ = 98.90594 u

13.28 Consider the D–T reaction (deuterium–tritium fusion)

$${}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + n$$

(a) Calculate the energy released in MeV in this reaction from the data:

$m({}^{2}_{1}H)$=2.014102 u

$m({}^{3}_{1}H)$ =3.016049 u

(b) Consider the radius of both deuterium and tritium to be approximately 2.0 fm. What is the kinetic energy needed to overcome the coulomb repulsion between the two nuclei? To what temperature must the gas be heated to initiate the reaction?

(Hint: Kinetic energy required for one fusion event =average thermal kinetic energy available with the interacting particles = $2(3kT/2)$; k = Boltzman's constant, T = absolute temperature.)

13.29 ** Obtain the maximum kinetic energy of β-particles, and the radiation frequencies of γ decays in the decay scheme shown in Fig. 13.6. You are given that

$m(^{198}Au) = 197.968233$ u

$m(^{198}Hg) = 197.966760$ u

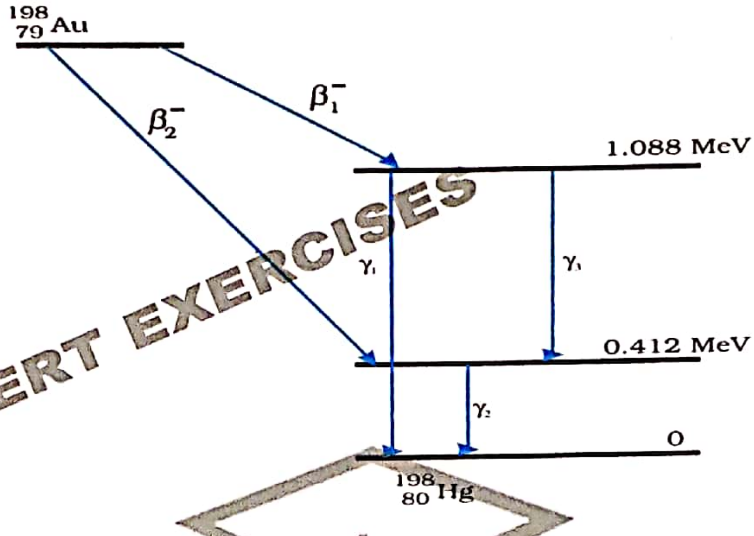

FIGURE13.6

13.30 Calculate and compare the energy released by a) fusion of 1.0 kg of hydrogen deep within Sun and b) the fission of 1.0 kg of ^{235}U in a fission reactor.

13.31 Suppose India had a target of producing by 2020 AD, 200,000 MW of electric power, ten percent of which was to be obtained from nuclear power plants. Suppose we are given that, on an average, the efficiency of utilization (i.e. conversion to electric energy) of thermal energy produced in a reactor was 25%. How much amount of fissionable uranium would our country need per year by 2020? Take the heat energy per fission of ^{235}U to be about 200MeV.