ANSWERS

Chapter 9

9.1 $v = -54$ cm. The image is real, inverted and magnified. The size of the image is 5.0 cm. As $u \to f$, $v \to \infty$; for $u < f$, image is virtual.

9.2 $v = 6.7$ cm. Magnification = 5/9, i.e., the size of the image is 2.5 cm. As $u \to \infty$; $v \to f$ (but never beyond) while $m \to 0$.

9.3 1.33; 1.7 cm

9.4 $n_{ga} = 1.51$; $n_{wa} = 1.32$; $n_{gw} = 1.144$; which gives sin r = 0.6181 i.e., $r \simeq 38°$.

9.5 $r = 0.8 \times \tan i_c$ and $\sin i_c = 1/1.33 \cong 0.75$, where r is the radius (in m) of the largest circle from which light comes out and i_c is the critical angle for water-air interface, Area = 2.6 m^2

9.6 $n \cong 1.53$ and D_m for prism in water $\cong 10°$

9.7 $R = 22$ cm

9.8 Here the object is virtual and the image is real. $u = +12$ cm (object on right; virtual)

(a) $f = +20$ cm. Image is real and at 7.5 cm from the lens on its right side.

(b) $f = -16$ cm. Image is real and at 48 cm from the lens on its right side.

9.9 $v = 8.4$ cm, image is erect and virtual. It is diminished to a size 1.8 cm. As $u \to \infty$, $v \to f$ (but never beyond f while $m \to 0$).

Note that when the object is placed at the focus of the concave lens (21 cm), the image is located at 10.5 cm (not at infinity as one might wrongly think).

9.10 A diverging lens of focal length 60 cm

9.11 (a) $v_e = -25$ cm and $f_e = 6.25$ cm give $u_e = -5$ cm; $v_O = (15 - 5)$ cm = 10 cm,

$f_O = u_O = -2.5$ cm; Magnifying power = 20

(b) $u_O = -2.59$ cm.

Magnifying power = 13.5.

9.12 Angular magnification of the eye-piece for image at 25 cm

$$= \frac{25}{2.5} + 1 = 11; \ |u_e| = \frac{25}{11}\text{cm} = 2.27\text{cm}; \ v_O = 7.2\text{cm}$$

Separation = 9.47 cm; Magnifying power = 88

9.13 24; 150 cm

9.14 (a) Angular magnification = 1500

(b) Diameter of the image = 13.7 cm.

9.15 Apply mirror equation and the condition:

(a) $f < 0$ (concave mirror); $u < 0$ (object on left)

(b) $f > 0$; $u < 0$

(c) $f > 0$ (convex mirror) and $u < 0$

(d) $f < 0$ (concave mirror); $f < u < 0$

to deduce the desired result.

9.16 The pin appears raised by 5.0 cm. It can be seen with an explicit ray diagram that the answer is independent of the location of the slab (for small angles of incidence).

9.17 (a) $\sin i'_c = 1.44/1.68$ which gives $i'_c = 59°$. Total internal reflection takes place when $i > 59°$ or when $r < r_{max} = 31°$. Now, $(\sin i_{max} / \sin r_{max}) = 1.68$, which gives $i_{max} \simeq 60°$. Thus, all incident rays of angles in the range $0 < i < 60°$ will suffer total internal reflections in the pipe. (If the length of the pipe is finite, which it is in practice, there will be a lower limit on i determined by the ratio of the diameter to the length of the pipe.)

(b) If there is no outer coating, $i'_c = \sin^{-1}(1/1.68) = 36.5°$. Now, $i = 90°$ will have $r = 36.5°$ and $i' = 53.5°$ which is greater than i'_c. Thus, *all* incident rays (in the range $53.5° < i < 90°$) will suffer total internal reflections.

9.18 (a) Rays converging to a point 'behind' a plane or convex mirror are reflected to a point in front of the mirror on a screen. In other words, a plane or convex mirror can produce a real image if the object is virtual. Convince yourself by drawing an appropriate ray diagram.

(b) When the reflected or refracted rays are divergent, the image is virtual. The divergent rays can be converged on to a screen by means of an appropriate converging lens. The convex lens of the eye does just that. The virtual image here serves as an object for the lens to produce a real image. Note, the screen here is not located at the position of the virtual image. There is no contradiction.

(c) Taller

(d) The apparent depth for oblique viewing decreases from its value for near-normal viewing. Convince yourself of this fact by drawing ray diagrams for different positions of the observer.

(e) Refractive index of a diamond is about 2.42, much larger than that of ordinary glass (about 1.5). The critical angle of diamond is about 24°, much less than that of glass. A skilled diamond-cutter exploits the larger range of angles of incidence (in the diamond), 24° to 90°, to ensure that light entering the diamond is totally reflected from many faces before getting out–thus producing a sparkling effect.

9.19 For fixed distance s between object and screen, the lens equation does not give a real solution for u or v if f is greater than $s/4$.

Therefore, $f_{max} = 0.75$ m.

9.20 21.4 cm

9.21 (a) (i) Let a parallel beam be the incident from the left on the convex lens first.

$f_1 = 30$ cm and $u_1 = -\infty$, give $v_1 = +30$ cm. This image becomes a virtual object for the second lens.

$f_2 = -20$ cm, $u_2 = +(30 - 8)$ cm $= +22$ cm which gives, $v_2 = -220$ cm. The parallel incident beam appears to diverge from a point 216 cm from the centre of the two-lens system.

(ii) Let the parallel beam be incident from the left on the concave lens first: $f_1 = -20$ cm, $u_1 = -\infty$, give $v_1 = -20$ cm. This image becomes a real object for the second lens: $f_2 = +30$ cm, $u_2 = -(20 + 8)$ cm $= -28$ cm which gives, $v_2 = -420$ cm. The parallel incident beam appears to diverge from a point 416 cm on the left of the centre of the two-lens system.

Clearly, the answer depends on which side of the lens system the parallel beam is incident. *Further we do not have a simple lens equation true for all u (and v) in terms of a definite constant of the system (the constant being determined by f_1 and f_2, and the separation between the lenses).* The notion of effective focal length, therefore, does not seem to be meaningful for this system.

(b) $u_1 = -40$ cm, $f_1 = 30$ cm, gives $v_1 = 120$ cm.

Magnitude of magnification due to the first (convex) lens is 3.
$u_2 = +(120 - 8)$ cm $= +112$ cm (object virtual);

$f_2 = -20$ cm which gives $v_2 = -\dfrac{112 \times 20}{92}$ cm

Magnitude of magnification due to the second (concave) lens = 20/92.

Net magnitude of magnification = 0.652

Size of the image = 0.98 cm

9.22 If the refracted ray in the prism is incident on the second face at the critical angle i_c, the angle of refraction r at the first face is $(60° - i_c)$.

Now, $i_c = \sin^{-1}(1/1.524) \simeq 41°$

Therefore, $r = 19°$

$\sin i = 0.4962$; $i \simeq 30°$

9.23 Two identical prisms made of the same glass placed with their bases on opposite sides (of the incident white light) and faces touching (or parallel) will neither deviate nor disperse, but will mearly produce a parallel displacement of the beam.

(a) To deviate without dispersion, choose, say, the first prism to be of crown glass, and take for the second prism a flint glass prism of suitably chosen refracting angle (smaller than that of crown glass prism because the flint glass prism disperses more) so that dispersion due to the first is nullified by the second.

(b) To disperse without deviation, increase the angle of flint glass prism (i.e., try flint glass prisms of greater and greater angle) so that deviations due to the two prisms are equal and opposite. (The flint glass prism angle will still be smaller than that of crown glass because flint glass has higher refractive index than that of crown glass). Because of the adjustments involved for so many colours, these are not meant to be precise arrangements for the purpose required.

9.24 To see objects at infinity, the eye uses its least converging power = (40+20) dioptres = 60 dioptres. This gives a rough idea of the distance between the retina and cornea-eye lens: (5/3) cm. To focus an object at the near point (u = –25 cm), on the retina (v = 5/3 cm), the focal length should be

$$\left[\frac{1}{25}+\frac{3}{5}\right]^{-1}=\frac{25}{16}\text{cm}$$

corresponding to a converging power of 64 dioptres. The power of the eye lens then is (64 – 40) dioptres = 24 dioptres. The range of accommodation of the eye-lens is roughly 20 to 24 dioptres.

9.25 No, a person may have normal ability of accommodation of the eye-lens and yet may be myopic or hypermetropic. Myopia arises when the eye-ball from front to back gets too elongated; hypermetropia arises when it gets too shortened. In practice, in addition, the eye lens may also lose some of its ability of accommodation. When the eyeball has the normal length but the eye lens loses partially its ability of accommodation (as happens with increasing age for any normal eye), the 'defect' is called presbyopia and is corrected in the same manner as hypermetropia.

9.26 The far point of the person is 100 cm, while his near point may have been normal (about 25 cm). Objects at infinity produce virtual image at 100 cm (using spectacles). To view closer objects i.e., those which are (or whose images using the spectacles are) between 100 cm and 25 cm, the person uses the ability of accommodation of his eye-lens. This ability usually gets partially lost in old age (presbyopia). The near point of the person recedes to 50 cm. To view objects at 25 cm clearly, the person needs converging lens of power +2 dioptres.

9.27 The defect (called astigmatism) arises because the curvature of the cornea plus eye-lens refracting system is not the same in different planes. [The eye-lens is usually spherical i.e., has the same curvature on different planes but the cornea is not spherical in case of an astigmatic eye.] In the present case, the curvature in the vertical plane is enough, so sharp images of vertical lines can be formed on the retina. But the curvature is insufficient in the horizontal plane, so horizontal lines appear blurred. The defect can be corrected by using a cylindrical lens with its axis along the vertical. Clearly, parallel rays in the vertical plane will suffer no extra refraction, but those in the horizontal plane can get the required extra convergence due to refraction by the curved surface of the cylindrical lens if the curvature of the cylindrical surface is chosen appropriately.

9.28 (a) Closest distance = $4\frac{1}{6}\text{cm} \approx 4.2\,\text{cm}$

Farthest distance = 5 cm

(b) Maximum angular magnification = [25/(25/6)] = 6. Minimum angular magnification = (25/5) = 5

9.29 (a) $\frac{1}{v}+\frac{1}{9}=\frac{1}{10}$

i.e., $v = -90$ cm,

Magnitude of magnification = 90/9 = 10.

Each square in the virtual image has an area $10 \times 10 \times 1\ \text{mm}^2 = 100\ \text{mm}^2 = 1\ \text{cm}^2$

(b) Magnifying power = 25/9 = 2.8

(c) No, magnification of an image by a lens and angular magnification (or magnifying power) of an optical instrument are two separate things. The latter is the ratio of the angular size of the object (which is equal to the angular size of the image even if the image is magnified) to the angular size of the object if placed at the near point (25 cm). Thus, magnification magnitude is $|(v/u)|$ and magnifying power is $(25/\ |u|)$. Only when the image is located at the near point $|v| = 25$ cm, are the two quantities equal.

9.30 (a) Maximum magnifying power is obtained when the image is at the near point (25 cm)

$u = -7.14$ cm.

(b) Magnitude of magnification = $(25/\ |u|) = 3.5$.

(c) Magnifying power = 3.5

Yes, the magnifying power (when the image is produced at 25 cm) is equal to the magnitude of magnification.

9.31 Magnification $= \sqrt{(6.25/1)} = 2.5$

$v = +2.5u$

$$+\frac{1}{2.5u} - \frac{1}{u} = \frac{1}{10}$$

i.e., $u = -6$ cm

$|v| = 15$ cm

The virtual image is closer than the normal near point (25 cm) and cannot be seen by the eye distinctly.

9.32 (a) Even though the absolute image size is bigger than the object size, the angular size of the image is equal to the angular size of the object. The magnifier helps in the following way: without it object would be placed no closer than 25 cm; with it the object can be placed much closer. The closer object has larger angular size than the same object at 25 cm. It is in this sense that angular magnification is achieved.

(b) Yes, it decreases a little because the angle subtended at the eye is then slightly less than the angle subtended at the lens. The effect is negligible if the image is at a very large distance away. [*Note:* When the eye is separated from the lens, the angles subtended at the eye by the first object and its image are not equal.]

(c) First, grinding lens of very small focal length is not easy. More important, if you decrease focal length, aberrations (both spherical and chromatic) become more pronounced. So, in practice, you cannot get a magnifying power of more than 3 or so with a simple convex lens. However, using an aberration corrected lens system, one can increase this limit by a factor of 10 or so.

(d) Angular magnification of eye-piece is $[(25/f_e) + 1]$ (f_e in cm) which increases if f_e is smaller. Further, magnification of the objective is given by

$$\frac{v_O}{|u_O|} = \frac{1}{(|u_O|/f_O) - 1}$$

which is large when $|u_O|$ is slightly greater than f_O. The microscope is used for viewing very close object. So $|u_O|$ is small, and so is f_O.

(e) The image of the objective in the eye-piece is known as 'eye-ring'. All the rays from the object refracted by objective go through the eye-ring. Therefore, it is an ideal position for our eyes for viewing. If we place our eyes too close to the eye-piece, we shall not collect much of the light and also reduce our field of view. If we position our eyes on the eye-ring and the area of the pupil of our eye is greater or equal to the area of the eye-ring, our eyes will collect all the light refracted by the objective. The precise location of the eye-ring naturally depends on the separation between the objective and the eye-piece. When you view through a microscope by placing your eyes on one end,the ideal distance between the eyes and eye-piece is usually built-in the design of the instrument.

9.33 Assume microscope in normal use i.e., image at 25 cm. Angular magnification of the eye-piece

$$= \frac{25}{5} + 1 = 6$$

Magnification of the objective

$$= \frac{30}{6} = 5$$

$$\frac{1}{5u_O} - \frac{1}{u_O} = \frac{1}{1.25}$$

which gives u_O= –1.5 cm; v_0= 7.5 cm. $|u_e|$= (25/6) cm = 4.17 cm. The separation between the objective and the eye-piece should be (7.5 + 4.17) cm = 11.67 cm. Further the object should be placed 1.5 cm from the objective to obtain the desired magnification.

9.34 (a) $m = (f_O/f_e) = 28$

(b) $m = \frac{f_O}{f_e}\left[1 + \frac{f_O}{25}\right] = 33.6$

9.35 (a) $f_O + f_e$ = 145 cm

(b) Angle subtended by the tower = (100/3000) = (1/30) rad.

Angle subtended by the image produced by the objective

$$= \frac{h}{f_O} = \frac{h}{140}$$

Equating the two, h = 4.7 cm.

(c) Magnification (magnitude) of the eye-piece = 6. Height of the final image (magnitude) = 28 cm.

9.36 The image formed by the larger (concave) mirror acts as virtual object for the smaller (convex) mirror. Parallel rays coming from the object at infinity will focus at a distance of 110 mm from the larger mirror. The distance of virtual object for the smaller mirror = (110 –20) = 90 mm. The focal length of smaller mirror is 70 mm. Using the mirror formula, image is formed at 315 mm from the smaller mirror.

9.37 The reflected rays get deflected by twice the angle of rotation of the mirror. Therefore, $d/1.5 = \tan 7°$. Hence d = 18.4 cm.

9.38 n = 1.33

CHAPTER 10

10.1 (a) Reflected light: (wavelength, frequency, speed same as incident light)

λ = 589 nm, $\nu = 5.09 \times 10^{14}$ Hz, $c = 3.00 \times 10^8$ m s^{-1}

(b) Refracted light: (frequency same as the incident frequency)
$\nu = 5.09 \times 10^{14}$Hz
$v = (c/n) = 2.26 \times 10^8$ m s^{-1}, $\lambda = (v/\nu)$ = 444 nm

10.2 (a) Spherical
(b) Plane
(c) Plane (a small area on the surface of a large sphere is nearly planar).

10.3 (a) 2.0×10^8 m s^{-1}
(b) No. The refractive index, and hence the speed of light in a medium, depends on wavelength. [When no particular wavelength or colour of light is specified, we may take the given refractive index to refer to yellow colour.] Now we know violet colour deviates more than red in a glass prism, i.e. $n_v > n_r$. Therefore, the violet component of white light travels slower than the red component.

10.4 $\lambda = \dfrac{1.2\times10^{-2}\times0.28\times10^{-3}}{4\times1.4}$ m = 600 nm

10.5 K/4

10.6 (a) 1.17 mm (b) 1.56 mm

10.7 0.15°

10.8 $\tan^{-1}(1.5) \simeq 56.3°$

10.9 5000 Å, 6×10^{14} Hz; 45°

10.10 40 m

10.11 Use the formula $\lambda' - \lambda = \dfrac{v}{c}\lambda$

i.e., $v = \dfrac{c}{\lambda}(\lambda' - \lambda) = \dfrac{3\times10^8\times15}{6563} = 6.86 \times 10^5$ m s^{-1}

10.12 In corpuscular (particle) picture of refraction, particles of light incident from a rarer to a denser medium experience a force of attraction normal to the surface. This results in an increase in the normal component of the velocity but the component along the surface is unchanged. This means

$c \sin i = v \sin r$ or $\dfrac{v}{c} = \dfrac{\sin i}{\sin r} = n$. Since $n > 1$, $v > c$.

The prediction is *opposite* to the experimental results ($v < c$). The wave picture of light is consistent with the experiment.

10.13 With the point object at the centre, draw a circle touching the mirror. This is a plane section of the spherical wavefront from the object that has just reached the mirror. Next draw the locations of this same wavefront after a time t in the presence of the mirror, and in the absence of the mirror. You will get two arcs symmetrically located on either side of the mirror. Using simple geometry, the centre of the reflected wavefront (the image of the object) is seen to be at the same distance from the mirror as the object.

10.14 (a) The speed of light in vacuum is a universal constant independent of all the factors listed and anything else. In particular, note the surprising fact that it is independent of the relative motion between the source and the observer. This fact is a basic axiom of Einstein's special theory of relativity.

(b) Dependence of the speed of light in a medium:

(i) does not depend on the nature of the source (wave speed is determined by the properties of the medium of propagation. This is also true for other waves, e.g., sound waves, water waves, etc.).

(ii) independent of the direction of propagation for *isotropic* media.

(iii) independent of the motion of the source relative to the medium but depends on the motion of the observer relative to the medium.

(iv) depends on wavelength.

(v) independent of intensity. [For high intensity beams, however, the situation is more complicated and need not concern us here.]

10.15 Sound waves require a medium for propagation. Thus even though the situations (i) and (ii) may correspond to the same relative motion (between the source and the observer), they are not identical physically since the motion of the observer *relative to the medium* is different in the two situations. Therefore, we cannot expect Doppler formulas for sound to be identical for (i) and (ii). For light waves in vacuum, there is clearly nothing to distinguish between (i) and (ii). Here only the relative motion between the source and the observer counts and the relativistic Doppler formula is the same for (i) and (ii). For light propagation in a medium, once again like for sound waves, the two situations are *not* identical and we should expect the Doppler formulas for this case to be different for the two situations (i) and (ii).

10.16 3.4×10^{-4} m.

10.17 (a) The size reduces by half according to the relation: size ~ λ/d. Intensity increases four fold.

(b) The intensity of interference fringes in a double-slit arrangement is modulated by the diffraction pattern of each slit.

(c) Waves diffracted from the edge of the circular obstacle interfere constructively at the centre of the shadow producing a bright spot.

(d) For diffraction or bending of waves by obstacles/apertures by a large angle, the size of the latter should be comparable to wavelength. If the size of the obstacle/aperture is much too large compared to wavelength, diffraction is by a small angle. Here the size is of the order of a few metres. The wavelength of light is about 5×10^{-7} m, while sound waves of, say, 1 kHz frequency have wavelength of about 0.3 m. Thus, sound waves can bend around the partition while light waves cannot.

(e) Justification based on what is explained in (d). Typical sizes of apertures involved in ordinary optical instruments are much larger than the wavelength of light.

10.18 12.5 cm.

10.19 0.2 nm.

10.20 (a) Interference of the direct signal received by the antenna with the (weak) signal reflected by the passing aircraft.

(b) Superposition principle follows from the linear character of the (differential) equation governing wave motion. If y_1 and y_2 are solutions of the wave equation, so is any linear combination of y_1 and y_2. When the amplitudes are large (e.g., high intensity laser beams) and non-linear effects are important, the situation is far more complicated and need not concern us here.

10.21 Divide the single slit into n smaller slits of width $a' = a/n$. The angle $\theta = n\lambda/a = \lambda/a'$. Each of the smaller slits sends zero intensity in the direction θ. The combination gives zero intensity as well.

Chapter 11

11.1 (a) 7.24×10^{18} Hz (b) 0.041 nm

11.2 (a) 0.34 eV = 0.54×10^{-19} J (b) 0.34 V (c) 344 km/s

11.3 1.5 eV = 2.4×10^{-19} J

11.4 (a) 3.14×10^{-19} J, 1.05×10^{-27} kg m/s (b) 3×10^{16} photons/s (c) 0.63 m/s

11.5 4×10^{21} photons/m^2 s

11.6 6.59×10^{-34} J s

11.7 (a) 3.38×10^{-19} J = 2.11 eV (b) 3.0×10^{20} photons/s

11.8 2.0 V

11.9 No, because $\nu < \nu_o$

11.10 4.73×10^{14} Hz

11.11 2.16 eV = 3.46×10^{-19} J

11.12 (a) 4.04×10^{-24} kg m s^{-1} (b) 0.164 nm

11.13 (a) 5.92×10^{-24} kg m s^{-1} (b) 6.50×10^{6} m s^{-1} (c) 0.112 nm

11.14 (a) 6.95×10^{-25} J = 4.34 μeV (b) 3.78×10^{-28} J = 0.236 neV

11.15 (a) 1.7×10^{-35} m (b) 1.1×10^{-32} m (c) 3.0×10^{-23} m

11.16 (a) 6.63×10^{-25} kg m/s (for both) (b) 1.24 keV (c) 1.51 eV

11.17 (a) 6.686×10^{-21} J = 4.174×10^{-2} eV (b) 0.145 nm

11.18 $\lambda = h/p = h/(h\nu/c) = c/\nu$

11.19 0.028 nm

11.20 (a) Use $eV = (m\,v^2/2)$ i.e., $v = [(2eV/m)]^{1/2}$; $v = 1.33 \times 10^{7}$ m s^{-1}.

(b) If we use the same formula with $V = 10^7$ V, we get $v = 1.88 \times 10^{9}$ m s^{-1}. This is clearly wrong, since nothing can move with a speed greater than the speed of light ($c = 3 \times 10^{8}$ m s^{-1}). Actually, the above formula for kinetic energy $(m\,v^2/2)$ is valid only when

$(v/c) << 1$. At very high speeds when (v/c) is comparable to (though always less than) 1, we come to the relativistic domain where the following formulae are valid:

Relativistic momentum $p = m\,v$

Total energy $E = m\,c^2$

Kinetic energy $K = m\,c^2 - m_0\,c^2$,

where the relativistic mass m is given by $m = m_0\left(1 - \frac{v^2}{c^2}\right)^{-1/2}$

m_0 is called the rest mass of the particle. These relations also imply:

$E = (p^2c^2 + m_0^2\,c^4)^{1/2}$

Note that in the relativisitc domain when v/c is comparable to 1, K or energy $\geq m_0c^2$ (rest mass energy). The rest mass energy of electron is about 0.51 MeV. Thus a kinetic energy of 10 MeV, being much greater than electron's rest mass energy, implies relativistic domain. Using relativistic formulas, v (for 10 MeV kinetic energy) = 0.999 c.

11.21 (a) 22.7 cm

(b) No. As explained above, a 20 MeV electron moves at relativistic speed. Consequently, the non-relativistic formula $R = (m_0 v/e\,B)$ is not valid. The relativistic formula is

$$R = p/eB = mv/eB \quad \text{or} \quad R = m_0 v/\left(eB\sqrt{1 - v^2/c^2}\right)$$

11.22 We have $e\,V = (m\,v^2/2)$ and $R = (m\,v/e\,B)$ which gives $(e/m) = (2V/R^2B^2)$; using the given data $(e/m) = 1.73 \times 10^{11}$ C kg^{-1}.

11.23 (a) 27.6 keV (b) of the order of 30 kV

11.24 Use $\lambda = (hc/E)$ with $E = 5.1 \times 1.602 \times 10^{-10}$J to get $\lambda = 2.43 \times 10^{-16}$ m .

11.25 (a) For $\lambda = 500$ m, $E = (h\,c/\,\lambda) = 3.98 \times 10^{-28}$J. Number of photons emitted per second

$= 10^4$J s^{-1}/3.98 × 10^{-28}J $\simeq 3 \times 10^{31}$ s^{-1}

We see that the energy of a radiophoton is exceedingly small, and the number of photons emitted per second in a radio beam is enormously large. There is, therefore, negligible error involved in ignoring the existence of a minimum quantum of energy (photon) and treating the total energy of a radio wave as continuous.

(b) For $\nu = 6 \times 10^{14}$ Hz, $E \simeq 4 \times 10^{-19}$J. Photon flux corresponding to minimum intensity

$= 10^{-10}$ W m^{-2}/4×10^{-19}J = 2.5 × 10^8 m^{-2} s^{-1}

Number of photons entering the pupil per second = 2.5 × 10^8 × 0.4 × 10^{-4} s^{-1} = 10^4s^{-1}. Though this number is not as large as in (a) above, it is large enough for us never to 'sense' or 'count' individual photons by our eye.

11.26 $\phi_0 = h\,\nu - e\,V_0 = 6.7 \times 10^{-19}$J = 4.2 eV; $\nu_0 = \frac{\phi_0}{h} = 1.0 \times 10^{15}$ Hz; λ = 6328Å corresponds to $\nu = 4.7 \times 10^{14}$ Hz $< \nu_0$. The photo-cell will not respond howsoever high be the intensity of laser light.

11.27 Use $e\,V_0 = h\,\nu - \phi_0$ for both sources. From the data on the first source, ϕ_0 = 1.40 eV. Use this value to obtain for the second source V_0 = 1.50 V.

11.28 Obtain V_0 versus ν plot. The slope of the plot is (h/e) and its intercept on the ν-axis is ν_0. The first four points lie nearly on a straight line which intercepts the ν-axis at $\nu_0 = 5.0 \times 10^{14}$ Hz (threshold frequency). The fifth point corresponds to $\nu < \nu_0$; there is no photoelectric emission and therefore no stopping voltage is required to stop the current. Slope of the plot is found to be 4.15×10^{-15} V s. Using $e = 1.6 \times 10^{-19}$ C, $h = 6.64 \times 10^{-34}$ J s (standard value $h = 6.626 \times 10^{-34}$ J s), $\phi_0 = h\,\nu_0 = 2.11$ V.

11.29 It is found that the given incident frequency ν is greater than ν_0 (Na), and ν_0 (K); but less than ν_0 (Mo), and ν_0 (Ni). Therefore, Mo and Ni will not give photoelectric emission. If the laser is brought closer, intensity of radiation increases, but this does not affect the result regarding Mo and Ni. However, photoelectric current from Na and K will increase in proportion to intensity.

11.30 Assume one conduction electron per atom. Effective atomic area $\sim 10^{-20}\,\text{m}^2$

Number of electrons in 5 layers

$$= \frac{5 \times 2 \times 10^{-4}\,\text{m}^2}{10^{-20}\,\text{m}^2} = 10^{17}$$

Incident power

$= 10^{-5}\,\text{W m}^{-2} \times 2 \times 10^{-4}\,\text{m}^2 = 2 \times 10^{-9}\,\text{W}$

In the wave picture, incident power is uniformly absorbed by all the electrons continuously. Consequently, energy absorbed per second per electron

$= 2 \times 10^{-9}/10^{17} = 2 \times 10^{-26}\,\text{W}$

Time required for photoelectric emission

$= 2 \times 1.6 \times 10^{-19}\,\text{J}/2 \times 10^{-26}\,\text{W} = 1.6 \times 10^{7}\,\text{s}$

which is about 0.5 year.

Implication: Experimentally, photoelectric emission is observed nearly instantaneously ($\sim 10^{-9}$ s): Thus, the wave picture is in gross disagreement with experiment. In the photon-picture, energy of the radiation is not continuously shared by all the electrons in the top layers. Rather, energy comes in discontinuous 'quanta'. and absorption of energy does not take place gradually. A photon is either not absorbed, or absorbed by an electron nearly instantly.

11.31 For $\lambda = 1$ Å, electron's energy = 150 eV; photon's energy = 12.4 keV. Thus, for the same wavelength, a photon has much greater energy than an electron.

11.32 (a) $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2\,mK}}$ Thus, for same K, λ decreases with m as $(1/\sqrt{m})$. Now $(m_n/m_e) = 1838.6$; therefore for the same energy, (150 eV) as in Ex. 11.31, wavelength of neutron = $\left(1/\sqrt{1838.6}\right) \times 10^{-10}$ m $= 2.33 \times 10^{-12}$ m. The interatomic spacing is about a hundred times greater. A neutron beam of 150 eV energy is therefore not suitable for diffraction experiments.

(b) $\lambda = 1.45 \times 10^{-10}$ m [Use $\lambda = (h/\sqrt{3\,m\,k\,T})$] which is comparable to interatomic spacing in a crystal.

Clearly, from (a) and (b) above, thermal neutrons are a suitable probe for diffraction experiments; so a high energy neutron beam should be first thermalised before using it for diffraction.

11.33 $\lambda = 5.5 \times 10^{-12}$ m

λ (yellow light) $= 5.9 \times 10^{-7}$m

Resolving Power (RP) is inversely proportional to wavelength. Thus, RP of an electron microscope is about 10^5 times that of an optical microscope. In practice, differences in other (geometrical) factors can change this comparison somewhat.

11.34 $p = \dfrac{h}{\lambda} = \dfrac{6.63 \times 10^{-34}\,\text{Js}}{10^{-15}\,\text{m}} = 6.63 \times 10^{-19}$ kg m s^{-1}

Use the relativistic formula for energy:

$E^2 = c^2p^2 + m_0^2\, c^4 = 9 \times (6.63)^2 \times 10^{-22} + (0.511 \times 1.6)^2 \times 10^{-26}$

$\simeq 9 \times (6.63)^2 \times 10^{-22}$,

the second term (rest mass energy) being negligible.

Therefore, $E = 1.989 \times 10^{-10}$ J $= 1.24$ BeV. Thus, electron energies from the accelerator must have been of the order of a few BeV.

11.35 Use $\lambda = \dfrac{h}{\sqrt{3\ m\ k\ T}}$; $m_{He} = \dfrac{4 \times 10^{3}}{6 \times 10^{23}}$kg

This gives $\lambda = 0.73 \times 10^{-10}$ m. Mean separation

$r = (V/N)^{1/3} = (kT/p)^{1/3}$

For $T = 300$ K, $p = 1.01 \times 10^5$ Pa, $r = 3.4 \times 10^{-9}$ m. We find $r >> \lambda$.

11.36 Using the same formula as in Exercise 11.35, $\lambda = 6.2 \times 10^{-9}$ m which is much greater than the given inter-electron separation.

11.37 (a) Quarks are thought to be confined within a proton or neutron by forces which grow stronger if one tries to pull them apart. It, therefore, seems that though fractional charges may exist in nature, observable charges are still integral multiples of *e*.

(b) Both the basic relations $e\,V = (1/2)\,m\,v^2$ or $e\,E = m\,a$ and $e\,B\,v = m\,v^2/r$, for electric and magnetic fields, respectively, show that the dynamics of electrons is determined not by *e*, and *m* separately but by the combination e/m.

(c) At low pressures, ions have a chance to reach their respective electrodes and constitute a current. At ordinary pressures, ions have no chance to do so because of collisions with gas molecules and recombination.

(d) Work function merely indicates the minimum energy required for the electron in the highest level of the conduction band to get out of the metal. Not all electrons in the metal belong to this level. They occupy a continuous band of levels. Consequently, for the same incident radiation, electrons knocked off from different levels come out with different energies.

(e) The absolute value of energy *E* (but not momentum *p*) of any particle is arbitrary to within an additive constant. Hence, while λ is physically significant, absolute value of ν of a matter wave of an electron has no direct physical meaning. The phase speed $\nu\lambda$ is likewise not physically significant. The group speed given by

$$\frac{d\nu}{d(1/\lambda)} = \frac{dE}{dp} = \frac{d}{dp}\left(\frac{p^2}{2m}\right) = \frac{p}{m}$$

is physically meaningful.

Chapter 12

12.1 (a) No different from

(b) Thomson's model; Rutherford's model

(c) Rutherford's model

(d) Thomson's model; Rutherford's model

(e) Both the models

12.2 The nucleus of a hydrogen atom is a proton. The mass of it is 1.67×10^{-27} kg, whereas the mass of an incident α-particle is 6.64×10^{-27} kg. Because the scattering particle is more massive than the target nuclei (proton), the α-particle won't bounce back in even in a head-on collision. It is similar to a football colliding with a tenis ball at rest. Thus, there would be no large-angle scattering.

12.3 820 nm.

12.4 5.6×10^{14} Hz

12.5 13.6 eV; –27.2 eV

12.6 9.7×10^{-8} m; 3.1×10^{15} Hz.

12.7 (a) 2.18×10^{6} m/s; 1.09×10^{6} m/s; 7.27×10^{5} m/s

(b) 1.52×10^{-16} s; 1.22×10^{-15} s; 4.11×10^{-15} s.

12.8 2.12×10^{-10} m; 4.77×10^{-10} m

12.9 Lyman series: 103 nm and 122 nm; Balmer series: 656 nm.

12.10 2.6×10^{74}

12.11 (a) About the same.

(b) Much less.

(c) It suggests that the scattering is predominantly due to a single collision, because the chance of a single collision increases linearly with the number of target atoms, and hence linearly with thickness.

(d) In Thomson's model, a single collision causes very little deflection. The observed average scattering angle can be explained only by considering multiple scattering. So it is wrong to ignore multiple scattering in Thomson's model. In Rutherford's model, most of the scattering comes through a single collision and multiple scattering effects can be ignored as a first approximation.

12.12 The first orbit Bohr's model has a radius a_0 given by

$a_0 = \dfrac{4\pi\varepsilon_0 (h/2\pi)^2}{m_e e^2}$. If we consider the atom bound by the gravitational force $(Gm_p m_e/r^2)$, we should replace $(e^2/4\pi\varepsilon_0)$ by $Gm_p m_e$. That is, the radius of the first Bohr orbit is given by $a_0^G = \dfrac{(h/2\pi)^2}{Gm_p m_e^2} \cong 1.2 \times 10^{29}$ m.

This is much greater than the estimated size of the whole universe!

12.13 $$\nu = \frac{me^4}{(4\pi)^3 \varepsilon_0^2 (h/2\pi)^3}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] = \frac{me^4(2n-1)}{(4\pi)^3 \varepsilon_0^2 (h/2\pi)^3 n^2 (n-1)^2}$$

For large n, $\nu \cong \dfrac{me^4}{32\pi^3 \varepsilon_0^2 (h/2\pi)^3 n^3}$

Orbital frequency $\nu_c = (v/2\pi r)$. In Bohr model $v = \dfrac{n(h/2\pi)}{mr}$, and

$r = \dfrac{4\pi\varepsilon_0 (h/2\pi)^2}{me^2} n^2$. This gives $\nu_c = \dfrac{n(h/2\pi)}{2\pi m r^2} = \dfrac{me^4}{32\pi^3 \varepsilon_0^2 (h/2\pi)^3 n^3}$

which is same as ν for large n.

12.14 (a) The quantity $\left(\dfrac{e^2}{4\pi\varepsilon_0 mc^2}\right)$ has the dimensions of length. Its value is 2.82×10^{-15}m – much smaller than the typical atomic size.

(b) The quantity $\dfrac{4\pi\varepsilon_0 (h/2\pi)^2}{me^2}$ has the dimensions of length. Its value is 0.53×10^{-10}m – of the order of atomic sizes.(Note that the dimensional arguments cannot, of course, tell us that we should use 4π and $h/2\pi$ in place of h to arrive at the right size.)

12.15 In Bohr's model, $mvr = nh$ and $\dfrac{mv^2}{r} = \dfrac{Ze^2}{4\pi\varepsilon_0 r^2}$

which give

$$T = \frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r}\ ;\ r = \frac{4\pi\varepsilon_0 \hbar^2}{Ze^2 m} n^2$$

These relations have nothing to do with the choice of the zero of potential energy. Now, choosing the zero of potential energy at infinity we have $V = -(Ze^2/4\pi\varepsilon_0 r)$ which gives $V = -2T$ and $E = T + V = -T$

(a) The quoted value of $E = -3.4$ eV is based on the customary choice of zero of potential energy at infinity. Using $E = -T$, the kinetic energy of the electron in this state is + 3.4 eV.

(b) Using $V = -2T$, potential energy of the electron is = – 6.8 eV

(c) If the zero of potential energy is chosen differently, kinetic energy does not change. Its value is + 3.4 eV independent of the choice of the zero of potential energy. The potential energy, and the total energy of the state, however, would alter if a different zero of the potential energy is chosen.

12.16 Angular momenta associated with planetary motion are incomparably large relative to h. For example, angular momentum of the earth in its orbital motion is of the order of $10^{70}h$. In terms of the Bohr's quantisation postulate, this corresponds to a very large value of n (of the order of 10^{70}). For such large values of n, the differences in the successive energies and angular momenta of the quantised levels of the Bohr model are so small compared to the energies and angular momenta respectively for the levels that one can, for all practical purposes, consider the levels continuous.

12.17 All that is needed is to replace m_e by m_μ in the formulas of the Bohr model. We note that keeping other factors fixed, $r \propto (1/m)$ and $E \propto m$. Therefore,

$$r_m = \frac{r_e m_e}{m_m} = \frac{0.53\times10^{\ 13}}{207} = 2.56 \times 10^{-13}\ \text{m}$$

$$E_\mu = \frac{E_e m_m}{m_e} = -\ (13.6 \times 207)\ \text{eV} \cong -\ 2.8\ \text{keV}$$

Chapter 13

13.1 (a) 6.941 u (b) 19.9%, 80.1%

13.2 20.18 u

13.3 104.7 MeV

13.4 8.79 MeV, 7.84 MeV

13.5 1.584×10^{25} MeV or 2.535×10^{12} J

13.6 i) ${}^{226}_{88}\mathrm{Ra} \rightarrow {}^{222}_{86}\mathrm{Rn} + {}^{4}_{2}\mathrm{He}$ ii) ${}^{242}_{94}\mathrm{Pu} \rightarrow {}^{238}_{92}\mathrm{U} + {}^{4}_{2}\mathrm{He}$

iii) ${}^{32}_{15}\mathrm{P} \rightarrow {}^{32}_{16}\mathrm{S} + \mathrm{e}^- + \bar{\nu}$ iv) ${}^{210}_{83}\mathrm{B} \rightarrow {}^{210}_{84}\mathrm{Po} + \mathrm{e}^- + \bar{\nu}$

v) ${}^{11}_{6}\mathrm{C} \rightarrow {}^{11}_{5}\mathrm{B} + \mathrm{e}^+ + \nu$ vi) ${}^{97}_{43}\mathrm{Tc} \rightarrow {}^{97}_{42}\mathrm{Mo} + \mathrm{e}^+ + \nu$

vii) ${}^{120}_{54}\mathrm{Xe} + \mathrm{e}^+ \rightarrow {}^{120}_{53}\mathrm{I} + \nu$

13.7 (a) 5 T years (b) 6.65 T years

13.8 4224 years

13.9 7.126×10^{-6} g

13.10 7.877×10^{10} Bq or 2.13 Ci

13.11 1.23

13.12 (a) Q = 4.93 MeV, E_α = 4.85 MeV (b) Q = 6.41 MeV, E_α = 6.29 MeV

13.13 ${}^{11}_{6}\mathrm{C} \rightarrow {}^{11}_{6}\mathrm{B} + \mathrm{e}^+ + \nu + Q$

$$Q = \left[m_N\left({}^{11}_{6}\mathrm{C}\right) - m_N\left({}^{11}_{6}\mathrm{B}\right) - m_e\right]c^2,$$

where the masses used are those of nuclei and not of atoms. If we use atomic masses, we have to add $6m_e$ in case of ${}^{11}\mathrm{C}$ and $5m_e$ in case of ${}^{11}\mathrm{B}$. Hence

$$Q = \left[m\left({}^{11}_{6}\mathrm{C}\right) - m\left({}^{11}_{6}\mathrm{B}\right) - 2m_e\right]c^2 \quad \text{(Note } m_e \text{ has been doubled)}$$

Using given masses, Q = 0.961 MeV.

$Q = E_d + E_e + E_\nu$

The daughter nucleus is too heavy compared to e^+ and ν, so it carries negligible energy ($E_d \approx 0$). If the kinetic energy (E_ν) carried by the neutrino is minimum (i.e., zero), the positron carries maximum energy, and this is practically all energy Q; hence maximum $E_e \approx Q$).

13.14 ${}^{23}_{10}\mathrm{Ne} \rightarrow {}^{23}_{11}\mathrm{Na} + \mathrm{e}^- + \bar{\nu} + Q$; $Q = \left[m_N\left({}^{23}_{10}\mathrm{Ne}\right) - m_N\left({}^{23}_{11}\mathrm{Na}\right) - m_e\right]c^2$, where the masses used are masses of nuclei and not of atoms as in Exercise 13.13. Using atomic masses $Q = \left[m\left({}^{23}_{10}\mathrm{Ne}\right) - m\left({}^{23}_{11}\mathrm{Na}\right)\right]c^2$. Note m_e has been cancelled. Using given masses, Q = 4.37 MeV. As in Exercise 13.13, maximum kinetic energy of the electron (max E_e) = Q = 4.37 MeV.

13.15 (i) Q = –4.03 MeV; endothermic

(ii) Q = 4.62 MeV; exothermic

13.16 $Q = m\left({}^{56}_{26}\mathrm{Fe}\right) - 2m\left({}^{28}_{13}\mathrm{Al}\right) = 26.90$ MeV; not possible.

13.17 4.536×10^{26} MeV

13.18 Energy generated per gram of ${}^{235}_{92}\mathrm{U}$ = $\dfrac{6 \times 10^{23} \times 200 \times 1.6 \times 10^{-13}}{235}\ \mathrm{J\,g^{-1}}$

The amount of $^{235}_{92}$U consumed in 5y with 80% on-time

$$= \frac{5 \times 0.8 \times 3.154 \times 10^{16} \times 235}{1.2 \times 1.6 \times 10^{13}} \text{ g} \qquad = 1544 \text{ kg}$$

The initial amount of $^{235}_{92}$U = 3088 kg.

13.19 About 4.9×10^4 y

13.20 360 KeV

13.22 Consider the competing processes:

$$^{A}_{Z}\text{X} \rightarrow {}^{A}_{Z-1}\text{Y} + \text{e}^{+} + \nu_e + Q_1 \quad \text{(positron capture)}$$

$$e^{-} + {}^{A}_{Z}\text{X} \rightarrow {}^{A}_{Z-1}\text{Y} + \nu_e + Q_2 \quad \text{(electron capture)}$$

$$Q_1 = \left[m_N\left({}^{A}_{Z}\text{X}\right) - m_N\left({}^{A}_{Z-1}\text{Y}\right) - m_e\right]c^2$$

$$= \left[m_N\left({}^{A}_{Z}\text{X}\right) - Z m_e - m\left({}^{A}_{Z-1}\text{Y}\right) - (Z-1)m_e - m_e\right]c^2$$

$$= \left[m\left({}^{A}_{Z}\text{X}\right) - m\left({}^{A}_{Z-1}\text{Y}\right) - 2m_e\right]c^2$$

$$Q_2 = \left[m_N\left({}^{A}_{Z}\text{X}\right) + m_e - m_N\left({}^{A}_{Z-1}\text{Y}\right)\right]c^2 = \left[m\left({}^{A}_{Z}\text{X}\right) - m\left({}^{A}_{Z-1}\text{Y}\right)\right]c^2$$

This means $Q_1 > 0$ implies $Q_2 > 0$ but $Q_2 > 0$ does not necessarily mean $Q_1 > 0$. Hence the result.

13.23 $^{25}_{12}\text{Mg}$: 9.3%, $^{26}_{12}\text{Mg}$: 11.7%

13.24 Neutron separation energy S_n of a nucleus $^{A}_{Z}\text{X}$ is

$$S_n = \left[m_N\left({}^{A-1}_{Z}\text{X}\right) + m_n - m_N\left({}^{A}_{Z}\text{X}\right)\right]c^2$$

From given data , $S_n({}^{41}_{20}\text{Ca}) = 8.36\text{MeV}, S_n({}^{27}_{13}\text{Al}) = 13.06\text{MeV}$

13.25 209 d

13.26 For $^{14}_{6}$C emission

$$Q = [m_N({}^{223}_{88}\text{Ra}) - m_N({}^{209}_{82}\text{Pb}) - m_N({}^{14}_{6}\text{C})]c^2$$

$$= [m({}^{223}_{88}\text{Ra}) - m({}^{209}_{82}\text{Pb}) - m({}^{14}_{6}\text{C})]c^2 = 31.85 \text{ MeV}$$

For $^{4}_{2}$He emission, $Q = [m({}^{223}_{88}\text{Ra}) - m({}^{219}_{86}\text{Rn}) - m({}^{4}_{2}\text{He})]c^2 = 5.98\text{MeV}$

13.27 $Q = [m({}^{238}_{92}\text{U}) + m_n - m({}^{140}_{58}\text{Ce}) - m({}^{99}_{44}\text{Ru})]c^2 = 231.1 \text{ MeV}$

13.28 (a) $Q = [m({}^{2}_{1}\text{H}) + m({}^{3}_{1}\text{H}) - m({}^{4}_{2}\text{He}) - m_n]c^2 = 17.59 \text{ MeV}$

(b) K.E. required to overcome Coulomb repulsion = 480.0 keV

480.0 KeV = 7.68×10^{-14} J = $3kT$

$$\therefore T = \frac{7.68 \times 10^{-14}}{3 \times 1.381 \times 10^{-23}} \qquad (as\ k = 1.381 \times 10^{-23} \text{ J K}^{-1})$$

= 1.85×10^9 K (required temperature)

13.29 $K_{max}\left(\beta_1^-\right) = 0.284 \text{ MeV}, K_{max}\left(\beta_2^-\right) = 0.960 \text{ MeV}$

$\nu(\gamma_1) = 2.627 \times 10^{20}$ Hz , $\nu(\gamma_2) = 0.995 \times 10^{20}$ Hz , $\nu(\gamma_3) = 1.632 \times 10^{20}$ Hz

13.30 (a) Note that in the interior of Sun, four ${}_1^1\text{H}$ nuclei combine to form one ${}_2^4\text{He}$ nucleus releasing about 26 MeV of energy per event. Energy released in fusion of 1kg of hydrogen = 39×10^{26} MeV

(b) Energy released in fission of 1kg of ${}_{92}^{235}\text{U} = 5.1 \times 10^{26}$ MeV

The energy released in fusion of 1 kg of hydrogen is about 8 times that of the energy released in the fusion of 1 kg of uranium.

13.31 3.076×10^4 kg

Chapter 14

14.1 (c)

14.2 (d)

14.3 (c)

14.4 (c)

14.5 (c)

14.6 (b), (c)

14.7 (c)

14.8 50 Hz for half-wave, 100 Hz for full-wave

14.9 $v_i = 0.01$ V ; $I_B = 10\ \mu$A

14.10 2V

14.11 No ($h\nu$ has to be greater than E_g).

14.12 $n_e \approx 4.95 \times 10^{22}$; $n_h = 4.75 \times 10^9$; n-type since $n_e >> n_h$

For charge neutrality $N_D - N_A = n_e - n_h$; $n_e.n_h = n_i^2$

Solving these equations, $n_e = \frac{1}{2}\left[(N_D - N_A) + \sqrt{(N_D - N_A)^2 + 4n_i^2}\right]$

14.13 About 1×10^5

14.14 (a) 0.0629 A, (b) 2.97 A, (c) 0.336 Ω

(d) For both the voltages, the current I will be almost equal to I_0, showing almost infinite dynamic resistance in the reverse bias.

14.16 NOT ;

A	Y
0	1
1	0

14.17 (a) AND (b) OR

14.18 OR gate

14.19 (a) NOT, (b) AND

Chapter 15

15.1 (b) 10 kHz cannot be radiated (antenna size), 1 GHz and 1000 GHz will penetrate.

15.2 (d) Consult Table 15.2

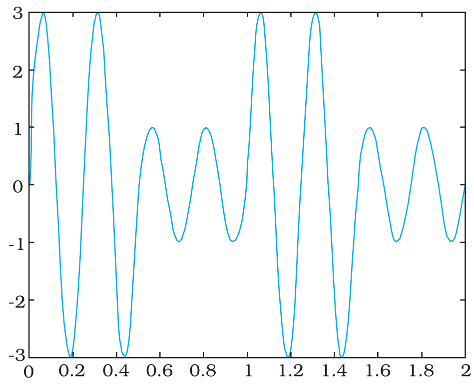
15.3 (c) Decimal system implies continuous set of values

15.4 No. Service area will be $A = \pi\ d_T^2 = \frac{22}{7} \times 162 \times 6.4 \times 10^6 = 3258\,\text{km}^2$.

15.5 $\mu = 0.75 = \frac{A_m}{A_c}$

$A_m = 0.75 \times 12 = 9\text{V}$.

15.6 (a)



(b) $\mu = 0.5$

15.7 Since the AM wave is given by $(A_c + A_m \sin\omega_m t) \cos \omega_c t$, the maximum amplitude is $M_1 = A_c + A_m$ while the minimum amplitude is $M_2 = A_c - A_m$. Hence the modulation index is

$$m = \frac{A_m}{A_c} = \frac{M_1 - M_2}{M_1 + M_2} = \frac{8}{12} = \frac{2}{3}.$$

With $M_2 = 0$, clearly, $m = 1$, irrespective of M_1.

15.8 Let, for simplicity, the received signal be
$A_1 \cos(\omega_c + \omega_m)\, t$
The carrier $A_c \cos \omega_c t$ is available at the receiving station. By multiplying the two signals, we get
$A_1 A_c \cos(\omega_c + \omega_a)\, t \cos \omega_c t$

$$= = \frac{A_1 A_c}{2}\left[\cos\left(2\omega_c + \omega_m\right)t + \cos\omega_m t\right]$$

If this signal is passed through a low-pass filter, we can record

the modulating signal $\frac{A_1 A_c}{2} \cos \omega_m t$.