NCERT EXERCISES 7.1

Find an anti derivative (or integral) of the following functions by the method of inspection.

1. $\sin 2x$
2. $\cos 3x$
3. e^{2x}
4. $(ax + b)^2$
5. $\sin 2x - 4e^{3x}$

Find the following integrals in Exercises 6 to 20:

6. $\int (4e^{3x} + 1)\, dx$
7. $\int x^2 (1 - \frac{1}{x^2})\, dx$
8. $\int (ax^2 + bx + c)\, dx$
9. $\int (2x^2 + e^x)\, dx$
10. $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$
11. $\int \frac{x^3 + 5x^2 - 4}{x^2}\, dx$
12. $\int \frac{x^3 + 3x + 4}{\sqrt{x}}\, dx$
13. $\int \frac{x^3 - x^2 + x - 1}{x - 1}\, dx$
14. $\int (1 - x)\sqrt{x}\, dx$
15. $\int \sqrt{x}(3x^2 + 2x + 3)\, dx$
16. $\int (2x - 3\cos x + e^x)\, dx$
17. $\int (2x^2 - 3\sin x + 5\sqrt{x})\, dx$
18. $\int \sec x (\sec x + \tan x)\, dx$
19. $\int \frac{\sec^2 x}{\text{cosec}^2 x}\, dx$
20. $\int \frac{2 - 3\sin x}{\cos^2 x}\, dx$

Choose the correct answer in Exercises 21 and 22.

21. The anti derivative of $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$ equals

(A) $\frac{1}{3}x^{\frac{1}{3}} + 2x^{\frac{1}{2}} + C$ (B) $\frac{2}{3}x^{\frac{2}{3}} + \frac{1}{2}x^2 + C$

(C) $\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$ (D) $\frac{3}{2}x^{\frac{2}{3}} + \frac{1}{2}x^{\frac{1}{2}} + C$

22. If $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$. Then $f(x)$ is

(A) $x^4 + \frac{1}{x^3} - \frac{129}{8}$ (B) $x^3 + \frac{1}{x^4} + \frac{129}{8}$

(C) $x^4 + \frac{1}{x^3} + \frac{129}{8}$ (D) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

EXERCISE 7.2

Integrate the functions in Exercises 1 to 37:

1. $\frac{2x}{1 + x^2}$
2. $\frac{(\log x)^2}{x}$
3. $\frac{1}{x + x\log x}$
4. $\sin x \sin(\cos x)$
5. $\sin(ax + b)\cos(ax + b)$
6. $\sqrt{ax + b}$
7. $x\sqrt{x + 2}$
8. $x\sqrt{1 + 2x^2}$
9. $(4x + 2)\sqrt{x^2 + x + 1}$
10. $\frac{1}{x - \sqrt{x}}$
11. $\frac{x}{\sqrt{x + 4}}, x > 0$
12. $(x^3 - 1)^{\frac{1}{3}} x^5$
13. $\frac{x^2}{(2 + 3x^3)^3}$
14. $\frac{1}{x(\log x)^m}, x > 0$

15. $\frac{x}{9-4x^2}$ 16. e^{2x+3} 17. $\frac{x}{e^{x^2}}$

18. $\frac{e^{\tan^{-1}x}}{1+x^2}$ 19. $\frac{e^{2x}-1}{e^{2x}+1}$ 20. $\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

21. $\tan^2(2x-3)$ 22. $\sec^2(7-4x)$ 23. $\frac{\sin^{-1}x}{\sqrt{1-x^2}}$

24. $\frac{2\cos x-3\sin x}{6\cos x+4\sin x}$ 25. $\frac{1}{\cos^2 x(1-\tan x)^2}$ 26. $\frac{\cos\sqrt{x}}{\sqrt{x}}$

27. $\sqrt{\sin 2x}\cos 2x$ 28. $\frac{\cos x}{\sqrt{1+\sin x}}$ 29. $\cot x \log \sin x$

30. $\frac{\sin x}{1+\cos x}$ 31. $\frac{\sin x}{(1+\cos x)^2}$ 32. $\frac{1}{1+\cot x}$

33. $\frac{1}{1-\tan x}$ 34. $\frac{\sqrt{\tan x}}{\sin x\cos x}$ 35. $\frac{(1+\log x)^2}{x}$

36. $\frac{(x+1)(x+\log x)^2}{x}$ 37. $\frac{x^3\sin(\tan^{-1}x^4)}{1+x^8}$

Choose the correct answer in Exercises 38 and 39.

38. $\int \frac{10x^9+10^x\log_{e^{10}}dx}{x^{10}+10^x}$ equals

(A) $10^x-x^{10}+C$ (B) $10^x+x^{10}+C$

(C) $(10^x-x^{10})^{-1}+C$ (D) $\log(10^x+x^{10})+C$

39. $\int \frac{dx}{\sin^2 x\cos^2 x}$ equals

(A) $\tan x+\cot x+C$ (B) $\tan x-\cot x+C$

(C) $\tan x\cot x+C$ (D) $\tan x-\cot 2x+C$

EXERCISE 7.3

Find the integrals of the functions in Exercises 1 to 22:

1. $\sin^2(2x+5)$ 2. $\sin 3x\cos 4x$ 3. $\cos 2x\cos 4x\cos 6x$

4. $\sin^3(2x+1)$ 5. $\sin^3 x\cos^3 x$ 6. $\sin x\sin 2x\sin 3x$

7. $\sin 4x\sin 8x$ 8. $\frac{1-\cos x}{1+\cos x}$ 9. $\frac{\cos x}{1+\cos x}$

10. $\sin^4 x$ 11. $\cos^4 2x$ 12. $\frac{\sin^2 x}{1+\cos x}$

13. $\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}$ 14. $\frac{\cos x-\sin x}{1+\sin 2x}$ 15. $\tan^3 2x\sec 2x$

16. $\tan^4 x$ 17. $\frac{\sin^3 x+\cos^3 x}{\sin^2 x\cos^2 x}$ 18. $\frac{\cos 2x+2\sin^2 x}{\cos^2 x}$

19. $\dfrac{1}{\sin x \cos^3 x}$ 20. $\dfrac{\cos 2x}{(\cos x + \sin x)^2}$ 21. $\sin^{-1}(\cos x)$

22. $\dfrac{1}{\cos(x-a)\cos(x-b)}$

Choose the correct answer in Exercises 23 and 24.

23. $\int \dfrac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x}\, dx$ is equal to

(A) $\tan x + \cot x + C$ (B) $\tan x + \operatorname{cosec} x + C$
(C) $-\tan x + \cot x + C$ (D) $\tan x + \sec x + C$

24. $\int \dfrac{e^x(1+x)}{\cos^2(e^x x)}\, dx$ equals

(A) $-\cot(ex^x) + C$ (B) $\tan(xe^x) + C$
(C) $\tan(e^x) + C$ (D) $\cot(e^x) + C$

EXERCISE 7.4

Integrate the functions in Exercises 1 to 23.

1. $\dfrac{3x^2}{x^6+1}$ 2. $\dfrac{1}{\sqrt{1+4x^2}}$ 3. $\dfrac{1}{\sqrt{(2-x)^2+1}}$

4. $\dfrac{1}{\sqrt{9-25x^2}}$ 5. $\dfrac{3x}{1+2x^4}$ 6. $\dfrac{x^2}{1-x^6}$

7. $\dfrac{x-1}{\sqrt{x^2-1}}$ 8. $\dfrac{x^2}{\sqrt{x^6+a^6}}$ 9. $\dfrac{\sec^2 x}{\sqrt{\tan^2 x+4}}$

10. $\dfrac{1}{\sqrt{x^2+2x+2}}$ 11. $\dfrac{1}{9x^2+6x+5}$ 12. $\dfrac{1}{\sqrt{7-6x-x^2}}$

13. $\dfrac{1}{\sqrt{(x-1)(x-2)}}$ 14. $\dfrac{1}{\sqrt{8+3x-x^2}}$ 15. $\dfrac{1}{\sqrt{(x-a)(x-b)}}$

16. $\dfrac{4x+1}{\sqrt{2x^2+x-3}}$ 17. $\dfrac{x+2}{\sqrt{x^2-1}}$ 18. $\dfrac{5x-2}{1+2x+3x^2}$

19. $\dfrac{6x+7}{\sqrt{(x-5)(x-4)}}$ 20. $\dfrac{x+2}{\sqrt{4x-x^2}}$ 21. $\dfrac{x+2}{\sqrt{x^2+2x+3}}$

22. $\dfrac{x+3}{x^2-2x-5}$ 23. $\dfrac{5x+3}{\sqrt{x^2+4x+10}}$.

Choose the correct answer in Exercises 24 and 25.

24. $\int \frac{dx}{x^2+2x+2}$ equals

(A) $x \tan^{-1}(x+1) + C$ (B) $\tan^{-1}(x+1) + C$

(C) $(x+1)\tan^{-1}x + C$ (D) $\tan^{-1}x + C$

25. $\int \frac{dx}{\sqrt{9x-4x^2}}$ equals

(A) $\frac{1}{9}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$ (B) $\frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)+C$

(C) $\frac{1}{3}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$ (D) $\frac{1}{2}\sin^{-1}\left(\frac{9x-8}{9}\right)+C$

EXERCISE 7.5

Integrate the rational functions in Exercises 1 to 21.

1. $\frac{x}{(x+1)(x+2)}$ 2. $\frac{1}{x^2-9}$ 3. $\frac{3x-1}{(x-1)(x-2)(x-3)}$

4. $\frac{x}{(x-1)(x-2)(x-3)}$ 5. $\frac{2x}{x^2+3x+2}$ 6. $\frac{1-x^2}{x(1-2x)}$

7. $\frac{x}{(x^2+1)(x-1)}$ 8. $\frac{x}{(x-1)^2(x+2)}$ 9. $\frac{3x+5}{x^3-x^2-x+1}$

10. $\frac{2x-3}{(x^2-1)(2x+3)}$ 11. $\frac{5x}{(x+1)(x^2-4)}$ 12. $\frac{x^3+x+1}{x^2-1}$

13. $\frac{2}{(1-x)(1+x^2)}$ 14. $\frac{3x-1}{(x+2)^2}$ 15. $\frac{1}{x^4-1}$

16. $\frac{1}{x(x^n+1)}$ [Hint: multiply numerator and denominator by x^{n-1} and put $x^n = t$]

17. $\frac{\cos x}{(1-\sin x)(2-\sin x)}$ [Hint : Put $\sin x = t$]

18. $\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$ 19. $\frac{2x}{(x^2+1)(x^2+3)}$ 20. $\frac{1}{x(x^4-1)}$

21. $\frac{1}{(e^x-1)}$ [Hint : Put $e^x = t$]

Choose the correct answer in each of the Exercises 22 and 23.

22. $\int \frac{x\,dx}{(x-1)(x-2)}$ equals

(A) $\log\left|\frac{(x-1)^2}{x-2}\right|+C$ (B) $\log\left|\frac{(x-2)^2}{x-1}\right|+C$

(C) $\log\left|\left(\frac{x-1}{x-2}\right)^2\right|+C$ (D) $\log|(x-1)(x-2)|+C$

23. $\int \frac{dx}{x(x^2+1)}$ equals

(A) $\log|x| - \frac{1}{2}\log(x^2+1) + C$ (B) $\log|x| + \frac{1}{2}\log(x^2+1) + C$

(C) $-\log|x| + \frac{1}{2}\log(x^2+1) + C$ (D) $\frac{1}{2}\log|x| + \log(x^2+1) + C$

EXERCISE 7.6

Integrate the functions in Exercises 1 to 22.

1. $x \sin x$ 2. $x \sin 3x$ 3. $x^2 e^x$ 4. $x \log x$
5. $x \log 2x$ 6. $x^2 \log x$ 7. $x \sin^{-1} x$ 8. $x \tan^{-1} x$
9. $x \cos^{-1} x$ 10. $(\sin^{-1} x)^2$ 11. $\frac{x \cos^{-1} x}{\sqrt{1-x^2}}$ 12. $x \sec^2 x$
13. $\tan^{-1} x$ 14. $x (\log x)^2$ 15. $(x^2 + 1) \log x$
16. $e^x (\sin x + \cos x)$ 17. $\frac{x e^x}{(1+x)^2}$ 18. $e^x \left(\frac{1+\sin x}{1+\cos x}\right)$
19. $e^x \left(\frac{1}{x} - \frac{1}{x^2}\right)$ 20. $\frac{(x-3) e^x}{(x-1)^3}$ 21. $e^{2x} \sin x$
22. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Choose the correct answer in Exercises 23 and 24.

23. $\int x^2 e^{x^3} dx$ equals

(A) $\frac{1}{3}e^{x^3} + C$ (B) $\frac{1}{3}e^{x^2} + C$

(C) $\frac{1}{2}e^{x^3} + C$ (D) $\frac{1}{2}e^{x^2} + C$

24. $\int e^x \sec x (1 + \tan x) dx$ equals

(A) $e^x \cos x + C$ (B) $e^x \sec x + C$

(C) $e^x \sin x + C$ (D) $e^x \tan x + C$

EXERCISE 7.7

Integrate the functions in Exercises 1 to 9.

1. $\sqrt{4-x^2}$ 2. $\sqrt{1-4x^2}$ 3. $\sqrt{x^2+4x+6}$
4. $\sqrt{x^2+4x+1}$ 5. $\sqrt{1-4x-x^2}$ 6. $\sqrt{x^2+4x-5}$
7. $\sqrt{1+3x-x^2}$ 8. $\sqrt{x^2+3x}$ 9. $\sqrt{1+\frac{x^2}{9}}$

Choose the correct answer in Exercises 10 to 11.

10. $\int \sqrt{1+x^2}\, dx$ is equal to

(A) $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|\left(x+\sqrt{1+x^2}\right)\right|+C$

(B) $\frac{2}{3}(1+x^2)^{\frac{3}{2}}+C$ (C) $\frac{2}{3}x(1+x^2)^{\frac{3}{2}}+C$

(D) $\frac{x^2}{2}\sqrt{1+x^2}+\frac{1}{2}x^2\log\left|x+\sqrt{1+x^2}\right|+C$

11. $\int\sqrt{x^2-8x+7}\,dx$ is equal to

(A) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}+9\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

(B) $\frac{1}{2}(x+4)\sqrt{x^2-8x+7}+9\log\left|x+4+\sqrt{x^2-8x+7}\right|+C$

(C) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-3\sqrt{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

(D) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

EXERCISE 7.8

Evaluate the following definite integrals as limit of sums.

1. $\int_a^b x\,dx$ 2. $\int_0^5(x+1)\,dx$ 3. $\int_2^3 x^2\,dx$

4. $\int_1^4(x^2-x)\,dx$ 5. $\int_{-1}^1 e^x\,dx$ 6. $\int_0^4(x+e^{2x})\,dx$

EXERCISE 7.9

Evaluate the definite integrals in Exercises 1 to 20.

1. $\int_{-1}^1(x+1)\,dx$ 2. $\int_2^3\frac{1}{x}\,dx$ 3. $\int_1^2(4x^3-5x^2+6x+9)\,dx$

4. $\int_0^{\frac{\pi}{4}}\sin 2x\,dx$ 5. $\int_0^{\frac{\pi}{2}}\cos 2x\,dx$ 6. $\int_4^5 e^x dx$ 7. $\int_0^{\frac{\pi}{4}}\tan x\,dx$

8. $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\operatorname{cosec} x\,dx$ 9. $\int_0^1\frac{dx}{\sqrt{1-x^2}}$ 10. $\int_0^1\frac{dx}{1+x^2}$ 11. $\int_2^3\frac{dx}{x^2-1}$

12. $\int_0^{\frac{\pi}{2}}\cos^2 x\,dx$ 13. $\int_2^3\frac{x\,dx}{x^2+1}$ 14. $\int_0^1\frac{2x+3}{5x^2+1}dx$ 15. $\int_0^1 x\,e^{x^2}dx$

16. $\int_1^2\frac{5x^2}{x^2+4x+3}$ 17. $\int_0^{\frac{\pi}{4}}(2\sec^2 x+x^3+2)\,dx$ 18. $\int_0^{\pi}(\sin^2\frac{x}{2}-\cos^2\frac{x}{2})\,dx$

19. $\int_0^2\frac{6x+3}{x^2+4}dx$ 20. $\int_0^1(x\,e^x+\sin\frac{\pi x}{4})\,dx$

Choose the correct answer in Exercises 21 and 22.

21. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ equals

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{12}$

22. $\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$ equals

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{12}$ (C) $\frac{\pi}{24}$ (D) $\frac{\pi}{4}$

EXERCISE 7.10

Evaluate the integrals in Exercises 1 to 8 using substitution.

1. $\int_0^1 \frac{x}{x^2+1} dx$ 2. $\int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\cos^5\phi\, d\phi$ 3. $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$

4. $\int_0^2 x\sqrt{x+2}$ (Put $x+2=t^2$) 5. $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx$

6. $\int_0^2 \frac{dx}{x+4-x^2}$ 7. $\int_{-1}^1 \frac{dx}{x^2+2x+5}$ 8. $\int_1^2 \left(\frac{1}{x}-\frac{1}{2x^2}\right) e^{2x} dx$

Choose the correct answer in Exercises 9 and 10.

9. The value of the integral $\int_{\frac{1}{3}}^1 \frac{(x-x^3)^{\frac{1}{3}}}{x^4} dx$ is

(A) 6 (B) 0 (C) 3 (D) 4

10. If $f(x) = \int_0^x t\sin t\, dt$, then $f'(x)$ is

(A) $\cos x + x\sin x$ (B) $x\sin x$

(C) $x\cos x$ (D) $\sin x + x\cos x$

EXERCISE 7.11

By using the properties of definite integrals, evaluate the integrals in Exercises 1 to 19.

1. $\int_0^{\frac{\pi}{2}} \cos^2 x\, dx$ 2. $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$ 3. $\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x\, dx}{\sin^{\frac{3}{2}} x+\cos^{\frac{3}{2}} x}$

4. $\int_0^{\frac{\pi}{2}} \frac{\cos^5 x\, dx}{\sin^5 x+\cos^5 x}$ 5. $\int_{-5}^5 |x+2|\, dx$ 6. $\int_2^8 |x-5|\, dx$

7. $\int_0^1 x(1-x)^n dx$ 8. $\int_0^{\frac{\pi}{4}} \log(1+\tan x)\, dx$ 9. $\int_0^2 x\sqrt{2-x}\, dx$

10. $\int_0^{\frac{\pi}{2}} (2\log\sin x - \log\sin 2x)\, dx$ 11. $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\, dx$

12. $\int_0^{\pi} \frac{x\,dx}{1+\sin x}$ 13. $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\,dx$ 14. $\int_0^{2\pi} \cos^5 x\,dx$

15. $\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1+\sin x \cos x}\,dx$ 16. $\int_0^{\pi} \log(1+\cos x)\,dx$ 17. $\int_0^{a} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\,dx$

18. $\int_0^4 |x-1|\,dx$

19. Show that $\int_0^a f(x)g(x)\,dx = 2\int_0^a f(x)\,dx$, if f and g are defined as $f(x) = f(a-x)$ and $g(x) + g(a-x) = 4$

Choose the correct answer in Exercises 20 and 21.

20. The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (x^3 + x\cos x + \tan^5 x + 1)\,dx$ is

(A) 0 (B) 2 (C) π (D) 1

21. The value of $\int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$ is

(A) 2 (B) $\frac{3}{4}$ (C) 0 (D) -2

Miscellaneous Exercise on Chapter 7

Integrate the functions in Exercises 1 to 24.

1. $\frac{1}{x-x^3}$ 2. $\frac{1}{\sqrt{x+a}+\sqrt{x+b}}$ 3. $\frac{1}{x\sqrt{ax-x^2}}$ [Hint: Put $x = \frac{a}{t}$]

4. $\frac{1}{x^2(x^4+1)^{\frac{3}{4}}}$ 5. $\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}$ [Hint: $\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}} = \frac{1}{x^{\frac{1}{3}}\left(1+x^{\frac{1}{6}}\right)}$, put $x = t^6$]

6. $\frac{5x}{(x+1)(x^2+9)}$ 7. $\frac{\sin x}{\sin(x-a)}$ 8. $\frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}}$

9. $\frac{\cos x}{\sqrt{4-\sin^2 x}}$ 10. $\frac{\sin^8 - \cos^8 x}{1-2\sin^2 x\cos^2 x}$ 11. $\frac{1}{\cos(x+a)\cos(x+b)}$

12. $\frac{x^3}{\sqrt{1-x^8}}$ 13. $\frac{e^x}{(1+e^x)(2+e^x)}$ 14. $\frac{1}{(x^2+1)(x^2+4)}$

15. $\cos^3 x\, e^{\log \sin x}$ 16. $e^{3\log x}(x^4+1)^{-1}$ 17. $f'(ax+b)\,[f(ax+b)]^n$

18. $\frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}}$ 19. $\frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}$, $x \in [0, 1]$

20. $\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$ 21. $\frac{2+\sin 2x}{1+\cos 2x}e^x$ 22. $\frac{x^2+x+1}{(x+1)^2(x+2)}$

23. $\tan^{-1}\sqrt{\frac{1-x}{1+x}}$ 24. $\frac{\sqrt{x^2+1}\left[\log(x^2+1)-2\log x\right]}{x^4}$

Evaluate the definite integrals in Exercises 25 to 33.

25. $\int_{\frac{\pi}{2}}^{\pi} e^x\left(\frac{1-\sin x}{1+\cos x}\right)dx$ 26. $\int_0^{\frac{\pi}{4}} \frac{\sin x\ \cos x}{\cos^4 x+\sin^4 x}\,dx$ 27. $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x\,dx}{\cos^2 x+4\sin^2 x}$

28. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x+\cos x}{\sqrt{\sin 2x}}\,dx$ 29. $\int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}}$ 30. $\int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{9+16\sin 2x}\,dx$

31. $\int_0^{\frac{\pi}{2}} \sin 2x\tan^{-1}(\sin x)\,dx$ 32. $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}\,dx$

33. $\int_1^4 [x-1|+|x-2|+|x-3|]\,dx$

Prove the following (Exercises 34 to 39)

34. $\int_1^3 \frac{dx}{x^2(x+1)}=\frac{2}{3}+\log\frac{2}{3}$ 35. $\int_0^1 x\,e^x dx=1$

36. $\int_{-1}^1 x^{17}\cos^4 x\,dx=0$ 37. $\int_0^{\frac{\pi}{2}} \sin^3 x\,dx=\frac{2}{3}$

38. $\int_0^{\frac{\pi}{4}} 2\tan^3 x\,dx=1-\log 2$ 39. $\int_0^1 \sin^{-1} x\,dx=\frac{\pi}{2}-1$

40. Evaluate $\int_0^1 e^{2-3x}dx$ as a limit of a sum.

Choose the correct answers in Exercises 41 to 44.

41. $\int \frac{dx}{e^x+e^{-x}}$ is equal to

(A) $\tan^{-1}(e^x)+C$ (B) $\tan^{-1}(e^{-x})+C$

(C) $\log(e^x-e^{-x})+C$ (D) $\log(e^x+e^{-x})+C$

42. $\int \frac{\cos 2x}{(\sin x+\cos x)^2}\,dx$ is equal to

(A) $\frac{-1}{\sin x+\cos x}+C$ (B) $\log|\sin x+\cos x|+C$

(C) $\log|\sin x-\cos x|+C$ (D) $\frac{1}{(\sin x+\cos x)^2}$

43. If $f(a+b-x)=f(x)$, then $\int_a^b x\,f(x)\,dx$ is equal to

(A) $\frac{a+b}{2}\int_a^b f(b-x)\,dx$ (B) $\frac{a+b}{2}\int_a^b f(b+x)\,dx$

(C) $\frac{b-a}{2}\int_a^b f(x)\,dx$ (D) $\frac{a+b}{2}\int_a^b f(x)\,dx$

44. The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$ is

(A) 1 (B) 0 (C) -1 (D) $\frac{\pi}{4}$