NCERT EXERCISES

Determine order and degree (if defined) of differential equations given in Exercises 1 to 10.

1. $\frac{d^4y}{dx^4} + \sin(y''') = 0$
2. $y' + 5y = 0$
3. $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$
4. $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$
5. $\frac{d^2y}{dx^2} = \cos 3x + \sin 3x$
6. $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$
7. $y''' + 2y'' + y' = 0$
8. $y' + y = e^x$
9. $y'' + (y')^2 + 2y = 0$
10. $y'' + 2y' + \sin y = 0$

11. The degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0 \text{ is}$$

(A) 3 (B) 2 (C) 1 (D) not defined

12. The order of the differential equation

$$2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0 \text{ is}$$

(A) 2 (B) 1 (C) 0 (D) not defined

EXERCISE 9.2

In each of the Exercises 1 to 10 verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation:

1. $y = e^x + 1$: $y'' - y' = 0$
2. $y = x^2 + 2x + C$: $y' - 2x - 2 = 0$
3. $y = \cos x + C$: $y' + \sin x = 0$
4. $y = \sqrt{1+x^2}$: $y' = \frac{xy}{1+x^2}$
5. $y = Ax$: $xy' = y \ (x \neq 0)$
6. $y = x\sin x$: $xy' = y + x\sqrt{x^2 - y^2}$ $(x \neq 0$ and $x > y$ or $x < -y)$
7. $xy = \log y + C$: $y' = \frac{y^2}{1-xy}$ $(xy \neq 1)$
8. $y - \cos y = x$: $(y\sin y + \cos y + x)\, y' = y$
9. $x + y = \tan^{-1}y$: $y^2 y' + y^2 + 1 = 0$
10. $y = \sqrt{a^2 - x^2}$ $x \in (-a, a)$: $x + y\frac{dy}{dx} = 0 \ (y \neq 0)$

11. The number of arbitrary constants in the general solution of a differential equation of fourth order are:

(A) 0 (B) 2 (C) 3 (D) 4

12. The number of arbitrary constants in the particular solution of a differential equation of third order are:

(A) 3 (B) 2 (C) 1 (D) 0

EXERCISE 9.3

In each of the Exercises 1 to 5, form a differential equation representing the given family of curves by eliminating arbitrary constants a and b.

1. $\frac{x}{a}+\frac{y}{b}=1$ 2. $y^2 = a(b^2 - x^2)$ 3. $y = a e^{3x} + b e^{-2x}$

4. $y = e^{2x}(a + bx)$ 5. $y = e^x(a \cos x + b \sin x)$

6. Form the differential equation of the family of circles touching the y-axis at origin.
7. Form the differential equation of the family of parabolas having vertex at origin and axis along positive y-axis.
8. Form the differential equation of the family of ellipses having foci on y-axis and centre at origin.
9. Form the differential equation of the family of hyperbolas having foci on x-axis and centre at origin.
10. Form the differential equation of the family of circles having centre on y-axis and radius 3 units.
11. Which of the following differential equations has $y = c_1 e^x + c_2 e^{-x}$ as the general solution?

(A) $\frac{d^2y}{dx^2}+y=0$ (B) $\frac{d^2y}{dx^2}-y=0$ (C) $\frac{d^2y}{dx^2}+1=0$ (D) $\frac{d^2y}{dx^2}-1=0$

12. Which of the following differential equations has $y = x$ as one of its particular solution?

(A) $\frac{d^2y}{dx^2}-x^2\frac{dy}{dx}+xy=x$ (B) $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+xy=x$

(C) $\frac{d^2y}{dx^2}-x^2\frac{dy}{dx}+xy=0$ (D) $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+xy=0$

EXERCISE 9.4

For each of the differential equations in Exercises 1 to 10, find the general solution:

1. $\frac{dy}{dx}=\frac{1-\cos x}{1+\cos x}$ 2. $\frac{dy}{dx}=\sqrt{4-y^2}\ (-2<y<2)$

3. $\frac{dy}{dx}+y=1\ (y\neq 1)$ 4. $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

5. $(e^x + e^{-x})\, dy - (e^x - e^{-x})\, dx = 0$ 6. $\frac{dy}{dx}=(1+x^2)(1+y^2)$

7. $y \log y\, dx - x\, dy = 0$ 8. $x^5\frac{dy}{dx}=-y^5$

9. $\frac{dy}{dx}=\sin^{-1}x$ 10. $e^x \tan y\, dx + (1 - e^x)\sec^2 y\, dy = 0$

For each of the differential equations in Exercises 11 to 14, find a particular solution satisfying the given condition:

11. $(x^3+x^2+x+1)\frac{dy}{dx}=2x^2+x;\ y=1$ when $x=0$

12. $x(x^2-1)\frac{dy}{dx}=1$; $y=0$ when $x=2$

13. $\cos\left(\frac{dy}{dx}\right)=a \quad (a \in \mathbf{R})$; $y=1$ when $x=0$

14. $\frac{dy}{dx}=y\tan x$; $y=1$ when $x=0$

15. Find the equation of a curve passing through the point (0, 0) and whose differential equation is $y' = e^x \sin x$.

16. For the differential equation $xy\frac{dy}{dx}=(x+2)(y+2)$, find the solution curve passing through the point (1, –1).

17. Find the equation of a curve passing through the point (0, –2) given that at any point (x, y) on the curve, the product of the slope of its tangent and y coordinate of the point is equal to the x coordinate of the point.

18. At any point (x, y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point (– 4, –3). Find the equation of the curve given that it passes through (–2, 1).

19. The volume of spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after t seconds.

20. In a bank, principal increases continuously at the rate of r% per year. Find the value of r if Rs 100 double itself in 10 years ($\log_e 2 = 0.6931$).

21. In a bank, principal increases continuously at the rate of 5% per year. An amount of Rs 1000 is deposited with this bank, how much will it worth after 10 years ($e^{0.5} = 1.648$).

22. In a culture, the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present?

23. The general solution of the differential equation $\frac{dy}{dx}=e^{x+y}$ is

(A) $e^x + e^{-y} = C$ (B) $e^x + e^y = C$

(C) $e^{-x} + e^y = C$ (D) $e^{-x} + e^{-y} = C$

EXERCISE 9.5

In each of the Exercises 1 to 10, show that the given differential equation is homogeneous and solve each of them.

1. $(x^2 + xy)\, dy = (x^2 + y^2)\, dx$
2. $y' = \frac{x+y}{x}$
3. $(x - y)\, dy - (x + y)\, dx = 0$
4. $(x^2 - y^2)\, dx + 2xy\, dy = 0$
5. $x^2\frac{dy}{dx} = x^2 - 2y^2 + xy$
6. $x\, dy - y\, dx = \sqrt{x^2 + y^2}\, dx$

7. $\left\{x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right\}y\,dx=\left\{y\sin\left(\frac{y}{x}\right)-x\cos\left(\frac{y}{x}\right)\right\}x\,dy$

8. $x\frac{dy}{dx}-y+x\sin\left(\frac{y}{x}\right)=0$

9. $y\,dx+x\log\left(\frac{y}{x}\right)dy-2x\,dy=0$

10. $\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$

For each of the differential equations in Exercises from 11 to 15, find the particular solution satisfying the given condition:

11. $(x+y)\,dy+(x-y)\,dx=0;\ y=1$ when $x=1$

12. $x^2\,dy+(xy+y^2)\,dx=0;\ y=1$ when $x=1$

13. $\left[x\sin^2\left(\frac{y}{x}\right)-y\right]dx+x\,dy=0;\ y=\frac{\pi}{4}$ when $x=1$

14. $\frac{dy}{dx}-\frac{y}{x}+\operatorname{cosec}\left(\frac{y}{x}\right)=0;\ y=0$ when $x=1$

15. $2xy+y^2-2x^2\frac{dy}{dx}=0;\ y=2$ when $x=1$

16. A homogeneous differential equation of the from $\frac{dx}{dy}=h\left(\frac{x}{y}\right)$ can be solved by making the substitution.

(A) $y=vx$ (B) $v=yx$ (C) $x=vy$ (D) $x=v$

17. Which of the following is a homogeneous differential equation?

(A) $(4x+6y+5)\,dy-(3y+2x+4)\,dx=0$

(B) $(xy)\,dx-(x^3+y^3)\,dy=0$

(C) $(x^3+2y^2)\,dx+2xy\,dy=0$

(D) $y^2\,dx+(x^2-xy-y^2)\,dy=0$

EXERCISE 9.6

For each of the differential equations given in Exercises 1 to 12, find the general solution:

1. $\frac{dy}{dx}+2y=\sin x$

2. $\frac{dy}{dx}+3y=e^{-2x}$

3. $\frac{dy}{dx}+\frac{y}{x}=x^2$

4. $\frac{dy}{dx}+\sec x\,y=\tan x\left(0\le x<\frac{\pi}{2}\right)$

5. $\cos^2 x\frac{dy}{dx}+y=\tan x\ \left(0\le x<\frac{\pi}{2}\right)$

6. $x\frac{dy}{dx}+2y=x^2\log x$

7. $x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x$

8. $(1+x^2)\,dy+2xy\,dx=\cot x\,dx\ (x\ne 0)$

9. $x\frac{dy}{dx}+y-x+xy\cot x=0\ (x\ne 0)$

10. $(x+y)\frac{dy}{dx}=1$

11. $y\,dx+(x-y^2)\,dy=0$

12. $(x+3y^2)\frac{dy}{dx}=y\ (y>0)$.

For each of the differential equations given in Exercises 13 to 15, find a particular solution satisfying the given condition:

13. $\frac{dy}{dx}+2y\tan x=\sin x;\ y=0$ when $x=\frac{\pi}{3}$

14. $(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{1+x^2};\ y=0$ when $x=1$

15. $\frac{dy}{dx}-3y\cot x=\sin 2x;\ y=2$ when $x=\frac{\pi}{2}$

16. Find the equation of a curve passing through the origin given that the slope of the tangent to the curve at any point (x, y) is equal to the sum of the coordinates of the point.

17. Find the equation of a curve passing through the point (0, 2) given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5.

18. The Integrating Factor of the differential equation $x\frac{dy}{dx}-y=2x^2$ is

(A) e^{-x} (B) e^{-y} (C) $\frac{1}{x}$ (D) x

19. The Integrating Factor of the differential equation

$(1-y^2)\frac{dx}{dy}+yx = ay\,(-1<y<1)$ is

(A) $\frac{1}{y^2-1}$ (B) $\frac{1}{\sqrt{y^2-1}}$ (C) $\frac{1}{1-y^2}$ (D) $\frac{1}{\sqrt{1-y^2}}$

Miscellaneous Exercise on Chapter 9

1. For each of the differential equations given below, indicate its order and degree (if defined).

(i) $\frac{d^2y}{dx^2}+5x\left(\frac{dy}{dx}\right)^2-6y=\log x$ (ii) $\left(\frac{dy}{dx}\right)^3-4\left(\frac{dy}{dx}\right)^2+7y=\sin x$

(iii) $\frac{d^4y}{dx^4}-\sin\left(\frac{d^3y}{dx^3}\right)=0$

2. For each of the exercises given below, verify that the given function (implicit or explicit) is a solution of the corresponding differential equation.

(i) $y = a\,e^{x} + b\,e^{-x} + x^2$: $x\frac{d^2y}{dx^2}+2\frac{dy}{dx}-xy+x^2-2=0$

(ii) $y = e^{x}(a\cos x + b\sin x)$: $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$

(iii) $y = x\sin 3x$: $\frac{d^2y}{dx^2}+9y-6\cos 3x=0$

(iv) $x^2 = 2y^2\log y$: $(x^2+y^2)\frac{dy}{dx}-xy=0$

3. Form the differential equation representing the family of curves given by $(x - a)^2 + 2y^2 = a^2$, where a is an arbitrary constant.

4. Prove that $x^2 - y^2 = c(x^2 + y^2)^2$ is the general solution of differential equation $(x^3 - 3xy^2)\,dx = (y^3 - 3x^2y)\,dy$, where c is a parameter.

5. Form the differential equation of the family of circles in the first quadrant which touch the coordinate axes.

6. Find the general solution of the differential equation $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$.

7. Show that the general solution of the differential equation $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$ is given by $(x + y + 1) = A(1 - x - y - 2xy)$, where A is parameter.

8. Find the equation of the curve passing through the point $\left(0, \frac{\pi}{4}\right)$ whose differential equation is $\sin x \cos y\, dx + \cos x \sin y\, dy = 0$.

9. Find the particular solution of the differential equation $(1 + e^{2x})\,dy + (1 + y^2)\,e^x\,dx = 0$, given that $y = 1$ when $x = 0$.

10. Solve the differential equation $y e^{\frac{x}{y}} dx = \left(x e^{\frac{x}{y}} + y^2\right) dy \; (y \neq 0)$.

11. Find a particular solution of the differential equation $(x - y)(dx + dy) = dx - dy$, given that $y = -1$, when $x = 0$. (Hint: put $x - y = t$)

12. Solve the differential equation $\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right]\frac{dx}{dy} = 1 \; (x \neq 0)$.

13. Find a particular solution of the differential equation $\frac{dy}{dx} + y\cot x = 4x \operatorname{cosec} x$ $(x \neq 0)$, given that $y = 0$ when $x = \frac{\pi}{2}$.

14. Find a particular solution of the differential equation $(x + 1)\frac{dy}{dx} = 2e^{-y} - 1$, given that $y = 0$ when $x = 0$.

15. The population of a village increases continuously at the rate proportional to the number of its inhabitants present at any time. If the population of the village was 20, 000 in 1999 and 25000 in the year 2004, what will be the population of the village in 2009?

16. The general solution of the differential equation $\frac{y\,dx - x\,dy}{y} = 0$ is

(A) $xy = C$ (B) $x = Cy^2$ (C) $y = Cx$ (D) $y = Cx^2$

17. The general solution of a differential equation of the type $\frac{dx}{dy} + P_1 x = Q_1$ is

(A) $y e^{\int P_1 dy} = \int \left(Q_1 e^{\int P_1 dy}\right) dy + C$

(B) $y . e^{\int P_1 dx} = \int \left(Q_1 e^{\int P_1 dx}\right) dx + C$

(C) $x e^{\int P_1 dy} = \int \left(Q_1 e^{\int P_1 dy}\right) dy + C$

(D) $x e^{\int P_1 dx} = \int \left(Q_1 e^{\int P_1 dx}\right) dx + C$

18. The general solution of the differential equation $e^x dy + (y e^x + 2x) dx = 0$ is

(A) $x e^y + x^2 = C$
(B) $x e^y + y^2 = C$
(C) $y e^x + x^2 = C$
(D) $y e^y + x^2 = C$